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Scattering of Two-Dimensional Electron Gas on the Semibounded Three-Dimensional Electron Gas

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From first principles, basing on the averaging of the equation for exact fluctuating distribution functions, the classical kinetic equation is derived for a two-dimensional electron gas (2DEG), interacting with a random potential of an external system. Using the approximation of a Fermi function with a shifted argument for the 2DEG distribution function the frequencies of energy and momentum relaxation are obtained. In the case of equilibrium external system the frequencies are expressed in terms of the dielectric functions of the 2DEG and external system. Energy and momentum relaxation frequencies of 2DEG, scattered on the three-dimensional electron gas, are calculated.

Из первых принципов, методом усреднения уравнений для точных флуктуирующих функций распределения, выведено классическое кинетическое уравнение для двумерного злектронного газа (2МЭГ), взаимодействующего со случайным потенциалом внешней системы. В приближении фермиевской функции со сдвинутым аргументом для функции распределения найдены частоты релаксации знергии и импульса 2МЭГ. В случае равновесной внешней системы частоты выражены через диэлектрические функции 2МЭГ и внешней системы. Рассчитаны частоты релаксации энергии и импульса 2МЭГ при рассеянии на полуограниченном трехмерном электронном газе.

1. Introduction

Electron–electron collisions in a closed system of charges cannot be responsible by themselves for energy and momentum relaxation, but have only an indirect influence on kinetic coefficients. The interaction between electrons, which belong to different systems and take different parts in energy and momentum transfer, can provide an efficient relaxation.

In this paper we consider the scattering of a two-dimensional electron gas (2DEG) on the semibounded three-dimensional electron gas (3DEG). Particles of the 3DEG occupy a halfspace separated by a distance l from the region of the 2DEG disposition. Such a situation occurs, for example, in heterojunctions and MIS structures, where 2DEG is separated by a depletion layer or oxide from the 3DEG. Another case is a thin layer of a narrow-gap semiconductor in a wide-gap one.

Intricate geometry of the system, presence of polar medium, and several kinds of mutually screening and scattering charge carriers makes the problem of complete and consistent allowance for all relevant effects nontrivial. Therefore, in Section 2 we derive the kinetic equation for 2DEG interacting with some external system. The equation is derived from first principles after the fashion of Klimontovich and Silin [1 to 3] and Rostoker [4]. In Section 3 we obtain the frequencies of 2DEG momentum and energy relaxation. For an equilibrium external system the frequencies can be expressed in a convenient form in terms of the

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dielectric function of the system. The latter is determined in Section 4 for a rather general model of an external system, in particular, that with the semibounded 3DEG. Calculation of the relaxation frequencies of the 2DEG scattering on 3DEG is performed in Section 5.

The 2DEG is supposed to occupy the lowest subband; transitions to other subbands are out of consideration. Both 2DEG and 3DEG are treated classically.

2. Kinetic Equation for 2DEG

For convenience in the beginning we consider an extremely thin layer of 2DEG ($z = 0$ plane). The generalization to the case of a finite extension ion z-direction will be given below.

We start with introducing the exact, fluctuating microscopic distribution function of the 2DEG,

$$\varrho(\boldsymbol{r}, \boldsymbol{v}, t) = \frac{1}{2}\left(\frac{2\pi\hbar}{m}\right)^2 \sum_i \delta(\boldsymbol{r} - \boldsymbol{r}_i(t))\, \delta(\boldsymbol{v} - \boldsymbol{v}_i(t)),$$

where $\boldsymbol{r} = (x, y)$, $\boldsymbol{r}_i(t)$ and $\boldsymbol{v}_i(t)$ being coordinate and velocity of the i-th individual particle, the sum is taken over all 2DEG particles. The function ϱ satisfies the equation

$$\frac{\partial \varrho}{\partial t} + \boldsymbol{v}\frac{\partial \varrho}{\partial \boldsymbol{r}} - \frac{e}{m}\left[\frac{\partial \varphi}{\partial \boldsymbol{r}} - \frac{1}{c}(\boldsymbol{v}\times\boldsymbol{H})\right]\frac{\partial \varrho}{\partial \boldsymbol{v}} = 0. \tag{1}$$

Here $\varphi(\boldsymbol{r}, t)$ is exact random potential, created by 2DEG particles and external system in $z = 0$ plane. We divide ϱ and φ into averaged (over a statistical ensemble) and fluctuating parts

$$\varrho(\boldsymbol{r}, \boldsymbol{v}, t) = f(\boldsymbol{r}, \boldsymbol{v}, t) + \delta\varrho(\boldsymbol{r}, \boldsymbol{v}, t), \qquad \varphi(\boldsymbol{r}, t) = \bar{\varphi}(\boldsymbol{r}, t) + \delta\varphi(\boldsymbol{r}, t).$$

Taking an average of (1) we obtain the kinetic equation for the 2DEG one-particle distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t) = \langle\varrho(\boldsymbol{r}, \boldsymbol{v}, t)\rangle$,

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}} - \frac{e}{m}\left[\frac{\partial \bar{\varphi}}{\partial \boldsymbol{r}} - \frac{1}{c}(\boldsymbol{v}\times\boldsymbol{H})\right]\frac{\partial f}{\partial \boldsymbol{v}} = \mathrm{St}\, f, \tag{2}$$

where the collisional term is

$$\mathrm{St}\, f = \frac{e}{m}\left\langle\frac{\partial \delta\varphi}{\partial \boldsymbol{r}}\frac{\partial \delta\varrho}{\partial \boldsymbol{v}}\right\rangle. \tag{3}$$

Subtracting (2) from (1) we find

$$\frac{\partial \delta\varrho}{\partial t} + \boldsymbol{v}\frac{\partial \delta\varrho}{\partial \boldsymbol{r}} - \frac{e}{m}\frac{\partial \delta\varphi}{\partial \boldsymbol{r}}\frac{\partial f}{\partial \boldsymbol{v}} = 0. \tag{4}$$

Writing down (4) we omitted the terms

$$-\frac{e}{m}\left[\frac{\partial \bar{\varphi}}{\partial \boldsymbol{r}} - \frac{1}{c}(\boldsymbol{v}\times\boldsymbol{H})\right]\frac{\partial \delta\varrho}{\partial \boldsymbol{v}} - \frac{e}{m}\left[\frac{\partial \delta\varphi}{\partial \boldsymbol{r}}\frac{\partial \delta\varrho}{\partial \boldsymbol{v}} - \left\langle\frac{\partial \delta\varphi}{\partial \boldsymbol{r}}\frac{\partial \delta\varrho}{\partial \boldsymbol{v}}\right\rangle\right],$$

corresponding to the direct influence of average external fields and collisions on the fluctuations (criteria of neglections are presented below).

Fluctuating part of the potential $\varphi(\boldsymbol{r}, t)$ can be split into the sum

$$\delta\varphi(\boldsymbol{r}, t) = \delta\varphi_{\mathrm{S}}(\boldsymbol{r}, t) + \delta\varphi_{\mathrm{e}}(\boldsymbol{r}, t).$$

Here $\delta\varphi_S$ is the fluctuation of the external system potential (e.g. that of impurities, phonons, other charge carriers) in the absence of 2DEG; $\delta\varphi_e$ is the potential of 2DEG particles, "dressed" by the potential induced in the polar medium. In the lowest order of perturbation theory the relation between $\delta\varphi_e$ and $\delta\varrho$ can be expressed within the linear response approximation, in Fourier representation this connection is specified by means of the dielectric function of the external system $\varepsilon_S(\omega, \boldsymbol{q})$,

$$\delta\varphi_e(\omega, \boldsymbol{q}) = \frac{2\pi e}{q\varepsilon_S(\omega, q)}\,\delta n(\omega, \boldsymbol{q})\,; \qquad \boldsymbol{q} = (q_x, q_y)\,, \tag{5}$$

where the density fluctuation is

$$\delta n(\boldsymbol{r}, t) = 2\left(\frac{m}{2\pi\hbar}\right)^2 \int \delta\varrho(\boldsymbol{r}, \boldsymbol{v}, t)\, \mathrm{d}^2\boldsymbol{v}\,. \tag{6}$$

The explicit form of $\varepsilon_S(\omega, \boldsymbol{q})$ is specified by the properties of the external system; for an unbounded system without spatial dispersion ε_S coincides with the dielectric permittivity of the medium.

Considering 2DEG as a homogeneous and quasistationary gas we perform the Fourier transform (one-side in time, two-side in space) over (4); temporarily we write $f(\boldsymbol{v})$ instead of $f(\boldsymbol{r}, \boldsymbol{v}, t)$. By solving the transformed equation (4) together with (5) and (6) we obtain

$$\delta\varrho(\omega, \boldsymbol{q}, \boldsymbol{v}) = \frac{i\,\delta\varrho(t = 0, \boldsymbol{q}, \boldsymbol{v})}{\omega - \boldsymbol{q}\boldsymbol{v} + i0} - \frac{e}{m}\,\frac{\delta\varphi(\omega, \boldsymbol{q})}{\omega - \boldsymbol{q}\boldsymbol{v} + i0}\,\boldsymbol{q}\,\frac{\partial f(\boldsymbol{v})}{\partial \boldsymbol{v}}\,, \tag{7}$$

$$\delta\varphi(\omega, \boldsymbol{q}) = \frac{4\pi i e}{q\varepsilon(\omega, \boldsymbol{q})}\left(\frac{m}{2\pi\hbar}\right)^2 \int \frac{\delta\varrho(t = 0, \boldsymbol{q}, \boldsymbol{v})}{\omega - \boldsymbol{q}\boldsymbol{v} + i0}\,\mathrm{d}^2\boldsymbol{v} + \frac{\varepsilon_S(\omega, \boldsymbol{q})}{\varepsilon(\omega, \boldsymbol{q})}\,\delta\varphi_S(\omega, \boldsymbol{q})\,, \tag{8}$$

where

$$\varepsilon(\omega, \boldsymbol{q}) = \varepsilon_S(\omega, \boldsymbol{q}) + \Delta\varepsilon_{2D}(\omega, \boldsymbol{q})\,; \tag{9}$$

$$\Delta\varepsilon_{2D}(\omega, \boldsymbol{q}) = \frac{e^2 m}{p\hbar^2 q} \int \frac{\boldsymbol{q}(\partial f/\partial \boldsymbol{v})}{\omega - \boldsymbol{q}\boldsymbol{v} + i0}\,\mathrm{d}^2\boldsymbol{v} \tag{10}$$

is dielectric permittivity of 2DEG [5].

Equations (7) and (8) are employed in the calculation of the collision integral (3). Taking the Fourier transform of (3) we find

$$\mathrm{St}\, f = -\frac{e}{(2\pi)^3\, m} \int \boldsymbol{q}\,\frac{\partial}{\partial \boldsymbol{v}}\, \mathrm{Im}\,\langle \delta\varphi\, \delta\varrho \rangle_{\omega, \boldsymbol{q}}\, \mathrm{d}\omega\, \mathrm{d}^2\boldsymbol{q}\,. \tag{11}$$

We set up the correlator of values (7) and (8), using for the correlator of initial functions $\delta\varrho(t = 0, \boldsymbol{q}, \boldsymbol{v})$ a form analogous to that of the three-dimensional case [5],

$$\langle \delta\varrho(t = 0, \boldsymbol{q}, \boldsymbol{v})\, \delta\varrho(t = 0, \boldsymbol{q}', \boldsymbol{v}')\rangle = (2\pi)^2\, \delta(\boldsymbol{q} + \boldsymbol{q}')\,\frac{2\pi^2\hbar^2}{m^2}\, f(\boldsymbol{v})\,[1 - f(\boldsymbol{v})]\,\delta(\boldsymbol{v} - \boldsymbol{v}')\,. \tag{12}$$

Finally, we get

$$\langle \delta\varphi\,\delta\varrho\rangle_{\omega,\boldsymbol{q}} = \frac{4\pi^2 e}{q\varepsilon(\omega,\boldsymbol{q})} f(\boldsymbol{v})\,[1 - f(\boldsymbol{v})]\,\delta(\omega - \boldsymbol{q}\boldsymbol{v})$$
$$+ \frac{4\pi e^3 m}{q^2\hbar^2\,|\varepsilon(\omega,\boldsymbol{q})|^2}\,\frac{1}{\boldsymbol{q}\boldsymbol{v} - \omega + i0}\,\boldsymbol{q}\frac{\partial f(\boldsymbol{v})}{\partial \boldsymbol{v}} \int \mathrm{d}^2\boldsymbol{v}' f(\boldsymbol{v}')\,[1 - f(\boldsymbol{v}')]\,\delta(\omega - \boldsymbol{q}\boldsymbol{v}')$$
$$+ \frac{e}{m}\,\frac{1}{\boldsymbol{q}\boldsymbol{v} - \omega + i0}\,\boldsymbol{q}\frac{\partial f(\boldsymbol{v})}{\partial \boldsymbol{v}}\,\frac{|\varepsilon_S(\omega,\boldsymbol{q})|^2}{|\varepsilon(\omega,\boldsymbol{q})|^2}\,\langle \delta\varphi_S^2\rangle_{\omega,\boldsymbol{q}}\,. \tag{13}$$

Substituting (13) into (11) we obtain the integral of collisions (see also [7, 8]). For convenience we divide it into two parts, relating to the mutual scattering of 2DEG particles and their scattering on the external system

$$\mathrm{St}\,f = \mathrm{St}_{ee}\,f + \mathrm{St}_{eS}\,f\,,$$
$$\mathrm{St}_{ee}\,f = \frac{e^4}{2\pi\hbar^2}\int \mathrm{d}^2\boldsymbol{q}\,\boldsymbol{q}\frac{\partial}{\partial \boldsymbol{v}}\left\{\frac{1}{q^2\,|\varepsilon(\boldsymbol{q}\boldsymbol{v},\boldsymbol{q})|^2}\int \mathrm{d}^2\boldsymbol{v}'\,\delta(\boldsymbol{q}\boldsymbol{v} - \boldsymbol{q}\boldsymbol{v}')\right.$$
$$\left.\times\left[\boldsymbol{q}\frac{\partial f(\boldsymbol{v})}{\partial \boldsymbol{v}} f(\boldsymbol{v}')\,[1 - f(\boldsymbol{v}')] - \boldsymbol{q}\frac{\partial f(\boldsymbol{v}')}{\partial \boldsymbol{v}'} f(\boldsymbol{v})\,[1 - f(\boldsymbol{v})]\right]\right\}, \tag{14}$$

$$\mathrm{St}_{eS}\,f = \frac{e^2}{2\pi m}\int \mathrm{d}^2\boldsymbol{q}\,\frac{\boldsymbol{q}}{q}\,\frac{\partial}{\partial \boldsymbol{v}}\left\{\frac{\mathrm{Im}\,\varepsilon_S(\boldsymbol{q}\boldsymbol{v},\boldsymbol{q})}{|\varepsilon(\boldsymbol{q}\bar{\boldsymbol{v}},\boldsymbol{q})|^2}\right.$$
$$\left.\times\left[f(\boldsymbol{v})\,[1 - f(\boldsymbol{v})] + \frac{\tilde{T}_S(\boldsymbol{q}\boldsymbol{v},\boldsymbol{q})}{(\boldsymbol{q}\boldsymbol{v})\,m}\,\boldsymbol{q}\frac{\partial f(\boldsymbol{v})}{\partial \boldsymbol{v}}\right]\right\}. \tag{15}$$

Here the effective temperature $\tilde{T}_S(\omega, \boldsymbol{q})$ is defined by the equation

$$\langle \delta\varphi_S^2\rangle_{\omega,\boldsymbol{q}} = \frac{4\pi\tilde{T}_S(\omega,\boldsymbol{q})}{\omega q}\,\frac{\mathrm{Im}\,\varepsilon_S(\omega,\boldsymbol{q})}{|\varepsilon_S(\omega,\boldsymbol{q})|^2}\,. \tag{16}$$

For an equilibrium external system with temperature T_S the effective temperature $\tilde{T}_S(\omega, \boldsymbol{q})$ coincides with T_S and (16) becomes the fluctuative–dissipative theorem.

It should be noticed that the kinetic equation (2) with collision integral given by (14), (15) has the Fokker-Planck form. The collision integral (14) is similar to that of Lehnard-Balescu for 3DEG [3, 6]. Equation (15) specifies the 2DEG scattering on the fluctuating potential of the external system. The screening in (14) and (15) is carried out both by 2DEG particles and by charges of the external system. The collision integral (14) is turned to zero by the Fermi function with shifted argument; for 2DEG and external system in equilibrium with each other, the Fermi function with temperature T_S turns to zero collision integral (15).

Now we allow for a finite thickness of 2DEG in z-direction; let $g(z)$ be the distribution function normalized by the condition $\int g(z)\,\mathrm{d}z = 1$. Thereby instead of potential $\varphi(\boldsymbol{r}, t, z = 0)$ the value $\varphi(\boldsymbol{r}, t) = \int \mathrm{d}z\, g(z)\,\varphi(\boldsymbol{r}, t, z)$ must be taken, whereas

$$\delta\varphi_e(\omega, \boldsymbol{q}, z) = \frac{2\pi e}{q}\,2\left(\frac{m}{2\pi\hbar}\right)^2 \int \frac{\mathrm{d}z'\, g(z')}{\varepsilon_S(\omega, \boldsymbol{q}, z, z')} \int \delta\varrho(\omega, \boldsymbol{q}, \boldsymbol{v})\,\mathrm{d}^2\boldsymbol{v}\,. \tag{17}$$

Here the dielectric function $\varepsilon_S(\omega, \boldsymbol{q}; z, z')$ specifies the response in the point z on the perturbation in z'.

As a result in above-stated equations one has

$$\frac{1}{\varepsilon_S(\omega, \boldsymbol{q})} = \int \mathrm{d}z \int dz' \frac{g(z)\, g(z')}{\varepsilon_S(\omega, \boldsymbol{q}, z, z')}, \tag{18}$$

$$\langle \delta\varphi_S^2 \rangle_{\omega, \boldsymbol{q}} = \int \mathrm{d}z \int \mathrm{d}z'\, g(z)\, g(z')\, \langle \delta\varphi_S(z)\, \delta\varphi_S(z') \rangle_{\omega, \boldsymbol{q}} \,. \tag{19}$$

Several remarks must be made upon the applicability of the obtained kinetic equations. Firstly, classical treatment of 2DEG and external system imposes restrictions on $\hbar\tilde{q}$ and $\hbar\tilde{\omega}$ – actual values of transferred momentum and energy,

$$\hbar\tilde{\omega} \equiv \hbar(\boldsymbol{q}\boldsymbol{v})_{\mathrm{act}} \ll T, T_S; \qquad \hbar\tilde{q} \ll mv\,, \tag{20}$$

where $v \approx \sqrt{W/m}$, W being the characteristic energy (thermal or Fermi) of 2DEG particles.

Furthermore, in the derivation of the collision integral the equation for fluctuations (4) was taken in fieldless and collisionless approximation. This requires the inequalities (for actual $\boldsymbol{q}$ and $\boldsymbol{v}$)

$$\boldsymbol{q}\boldsymbol{v},\, q_s v \gg \nu(\boldsymbol{q}, \boldsymbol{v}),\, \omega_E,\, \omega_H\,, \tag{21}$$

where $q_s = 2\pi e^2 n/W$ is the inverse screening length in 2DEG [5], n the 2DEG concentration, $\omega_E = eE/mv$, $\omega_H = eH/mc$.

The frequency $\nu(\boldsymbol{q}, \boldsymbol{v})$ of collisions with momentum transfer $\hbar\boldsymbol{q}$ is evaluated as $\mathrm{St}\, f(\boldsymbol{q}, \boldsymbol{v}) \approx \nu(\boldsymbol{q}, \boldsymbol{v})\, \delta f(\boldsymbol{q}, \boldsymbol{v})$ and has the form

$$\nu = \nu_{ee} + \nu_{eS} = \frac{e^2 q}{mv} \frac{\mathrm{Im}\, \Delta\varepsilon_{2D}(\boldsymbol{q}\boldsymbol{v}, \boldsymbol{q}) + \mathrm{Im}\, \varepsilon_S(\boldsymbol{q}\boldsymbol{v}, \boldsymbol{q})}{|\varepsilon(\boldsymbol{q}\boldsymbol{v}, \boldsymbol{q}|^2}\,.$$

Here ν_{eS} is the frequency of 2DEG particle collisions with the external system. The frequency of electron–electron collisions is $\nu_{ee} \approx e^4 nq/m^{1/2} W^{3/2}\, |\varepsilon|^2$. Insertion of ν_{ee} in (28) gives two inequalities,

$$e^4 n/W^2\, |\varepsilon|^2 \ll 1\,, \qquad q^{-1} \gg e^2/|\varepsilon|\, W\,.$$

The first inequality requires the mean potential energy to be small as compared to the mean kinetic energy (weakly nonideal 2D plasma). The second one restricts the consideration by distance, where the potential energy of interaction between the particles is less than the kinetic energy (Born approximation).

3. Frequencies of Momentum and Energy Relaxation

Employing (2) with collision integral (14), (15) we write down the momentum and energy balance equations.

Introducing velocity averaging by

$$\langle\!\langle A(\boldsymbol{v}) \rangle\!\rangle = \frac{2}{n(\boldsymbol{r}, t)} \left(\frac{m}{2\pi\hbar}\right)^2 \int \mathrm{d}^2\boldsymbol{v}\, f(\boldsymbol{r}, \boldsymbol{v}, t)\, A(\boldsymbol{v})\,,$$

we define average characteristic of 2DEG – current density $\boldsymbol{j} = en \langle\!\langle \boldsymbol{v} \rangle\!\rangle$, momentum flux density $\pi_{ij} = mn \langle\!\langle v_i v_j \rangle\!\rangle$, kinetic energy density $\mathscr{E} = n \langle\!\langle mv^2/2 \rangle\!\rangle$, flux of kinetic energy density $\boldsymbol{Q} = n \langle\!\langle \boldsymbol{v} mv^2/2 \rangle\!\rangle$.

The balance equations have the form

$$\frac{\partial \boldsymbol{j}}{\partial t} + \frac{e}{m}\nabla\hat{\pi} = \frac{en}{m}\left[e\boldsymbol{E} + \frac{1}{nc}(\boldsymbol{j}\times\boldsymbol{H}) - \mathscr{R}\right], \tag{22}$$

$$\frac{\partial \mathscr{E}}{\partial t} + \operatorname{div}\boldsymbol{Q} = \boldsymbol{j}\cdot\boldsymbol{E} - n\mathscr{P}. \tag{23}$$

Here $-\mathscr{R} = m\langle\langle \boldsymbol{v} f^{-1}\,\mathrm{St_{eS}}\, f\rangle\rangle$ is a friction force against medium, $\mathscr{P} = -(m/2) \times\langle\langle v^2 f^{-1}\,\mathrm{St_{eS}}\, f\rangle\rangle$ is a power transferred (per particle) to the external system. Interaction between 2DEG particles, specified by (14), does not contribute to $\mathscr{R}$ and $\mathscr{P}$.

With help of (15) the equations for $\mathscr{R}$ and $\mathscr{P}$ are reduced to the form

$$\begin{pmatrix}\mathscr{R}\\ \mathscr{P}\end{pmatrix} = \frac{e^2}{\pi n}\left(\frac{m}{2\pi\hbar}\right)^2 \int\frac{\mathrm{d}^2\boldsymbol{q}}{q}\int \mathrm{d}^2\boldsymbol{v}\begin{pmatrix}\boldsymbol{q}\\ \boldsymbol{q}\boldsymbol{v}\end{pmatrix}\frac{\operatorname{Im}\varepsilon_{\mathrm{S}}(\boldsymbol{q}\boldsymbol{v},\boldsymbol{q})}{|\varepsilon(\boldsymbol{q}\boldsymbol{v},\boldsymbol{q})|^2} \times\left\{f(\boldsymbol{v})\,[1-f(\boldsymbol{v})] + \frac{\tilde{T}_{\mathrm{S}}(\boldsymbol{q}\boldsymbol{v},\boldsymbol{q})}{(\boldsymbol{q}\boldsymbol{v})\,m}\,\boldsymbol{q}\frac{\partial f(\boldsymbol{v})}{\partial \boldsymbol{v}}\right\}. \tag{24}$$

To evaluate $\mathscr{P}$ in (24) we employ a model function $f_0(\boldsymbol{v})$ of Fermi form with some temperature T. For an equilibrium external system that yields $\mathscr{P} = \nu_\varepsilon(T - T_{\mathrm{S}})$. Applying a Fermi function with shifted argument $f_0(\boldsymbol{v} - \boldsymbol{u})$ to the equation for $\mathscr{R}$, after linearization in $\boldsymbol{u}$ we can present $\mathscr{R}$ in the form $\mathscr{R}_i = \nu^m_{ij} m u_j$. Linearization is valid if the drift velocity u is small compared with the characteristic velocity of 2DEG particles. A sufficient condition would be $(T - T_{\mathrm{S}})/T \ll \nu_p/\nu_\varepsilon$.

For the dielectric function $\varepsilon(\omega, q)$, depending on $|\boldsymbol{q}|$ only, we have $\nu^m_{ij} = \nu_m\delta_{ij}$ and

$$\begin{pmatrix}\nu_m\\ \nu_\varepsilon\end{pmatrix} = \frac{1}{\pi^2 n}\int_0^\infty\frac{\mathrm{d}\omega}{\omega^2}\int_0^\infty q\,dq\begin{pmatrix}\omega^2\\ Tq^2/2m\end{pmatrix}\frac{\operatorname{Im}\varepsilon_{\mathrm{S}}(\omega,q)\operatorname{Im}\Delta\varepsilon^{\mathrm{eq}}_{2\mathrm{D}}(\omega,q)}{|\varepsilon(\omega,q)|^2}. \tag{25}$$

The superscript "eq" indicates that the value $\Delta\varepsilon^{\mathrm{eq}}$ must be calculated with an equilibrium distribution function.

For a stationary and homogeneous system in the absence of a magnetic field the energy balance equation takes the form $jE = n\nu_\varepsilon(T - T_{\mathrm{S}})$. Hence it follows

$$T = T_{\mathrm{S}}(1 + e^2E^2/mT_{\mathrm{S}}\nu_m\nu_\varepsilon).$$

It should be mentioned that the model function f_0 used in the derivation of (25) is an exact solution of the kinetic equation in the limit of strong e–e scattering ($\nu^{m,\varepsilon}_{\mathrm{eS}} \ll \nu_{\mathrm{ee}}$ in the actual region of energy). In many cases (excluding the well-known peculiarities of optical phonon scattering [9]) even in the opposite limite of $\nu_{\mathrm{eS}} \gg \nu_{\mathrm{ee}}$, application of model functions gives results that do not differ much from the exact ones [10].

4. Dielectric Function of the System

In this section we calculate the function $\varepsilon_{\mathrm{S}}(\omega, \boldsymbol{q})$ for a model system of a rather general form, applicable to the cases of single heterojunction or MIS structure (Fig. 1a) and double heterojunction (Fig. 1b). Values ε_-, ε_+, and ε_0 are dielectric permittivities in the corresponding regions; 2DEG adjoins the $z = 0$ plane on the right (Fig. 1a) or is confined in the $0 < z < \lambda$ layer; 3DEG occupies the halfspace $z < -l$. At first we shall not specify the

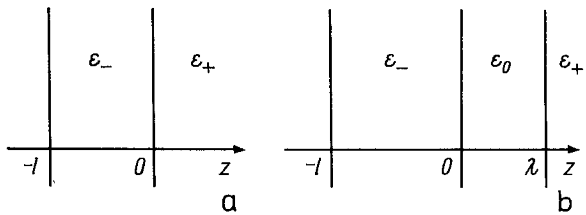

Fig. 1. Model of a) single heterojunction or MIS structure and b) double heterojunction

properties of the medium in the $z < -l$ region (impurities, 3DEG, metal electrode, another heterojunction, etc. can be disposed here). The influence of this medium is formally taken into account by a boundary condition of general form, relating the fluctuating parts of induction and potential,

$$\delta D_z(\omega, \boldsymbol{q}, z = -l + 0) = \frac{-\varepsilon_- q\, \delta\varphi(\omega, \boldsymbol{q}, z = -l + 0)}{\beta(\omega, \boldsymbol{q})}. \tag{26}$$

Here the function $\beta(\omega, \boldsymbol{q})$ pattern is set by the properties of the medium in the $z < -l$ region. For instance, in the case of metal electrode in the $z = -l$ plane we have $\beta = 0$; for a medium homogeneous in the $z < 0$ region $\beta = 1$.

In accordance with (18), the dielectric function $\varepsilon_S(\omega, \boldsymbol{q})$ is specified by the 2DEG density function $g(z)$ and dielectric function of inhomogeneous system $\varepsilon_S(\omega, \boldsymbol{q}, z, z')$ in the region, occupied by 2DEG. For calculation of $\varepsilon_S(\omega, \boldsymbol{q}, z, z')$ one must find the potential created in the z-plane by the source, located in the z'-plane (cf. (17)). The result for model b is $(0 < z < z' < \lambda)$

$$\varepsilon_S(\omega, \boldsymbol{q}, z, z') = \frac{\varepsilon_0}{2} \frac{\varepsilon_0(\varepsilon_+ + \tilde{\varepsilon}_-)\cosh q\lambda + (\varepsilon_0^2 + \varepsilon_+\tilde{\varepsilon}_-)\sinh q\lambda}{(\varepsilon_0 \cosh qz + \tilde{\varepsilon}_- \sinh qz)\,[\varepsilon_0 \cosh q(\lambda - z') + \varepsilon_+ \sinh q(\lambda - z')]}, \tag{27}$$

where

$$\tilde{\varepsilon}_- \equiv \tilde{\varepsilon}_-(\omega, \boldsymbol{q}) = \frac{\beta(\omega, q)\tanh ql + 1}{\beta(\omega, q) + \tanh ql}\,\varepsilon_- \tag{28}$$

stands for the effective dielectric permittivity of the medium in the $z < -l$ region; $\varepsilon_S(\omega, \boldsymbol{q}, z, z')|_{z'<z} = \varepsilon_S(\omega, \boldsymbol{q}, z', z)|_{z<z'}$. The equation for model a can be obtained from (27) by taking the limit $\lambda \to \infty$ and substituting ε_+ for ε_0 (see also [11].

By making use of (18) and the variation density function [12] of a single heterojunction $g(z) = b^3 z^2 \exp(-bz)/2$, we find

$$\varepsilon_S(\omega, \boldsymbol{q}) = \frac{\varepsilon_+(\varepsilon_+ + \tilde{\varepsilon}_-)(1 + q/b)^6}{\varepsilon_+ - \tilde{\varepsilon}_- + (\varepsilon_+ + \tilde{\varepsilon}_-)(1 + q/b)^3(1 + 9q/8b + 3q^2/8b^2)}. \tag{29}$$

In the case of $\beta = 1$ (29) goes over into the corresponding result of [5].

In the case of a double heterojunction with square potential well we use $g(z) = (2/\lambda)\sin^2(\pi z/\lambda)$. Insertion of (27) into (18) gives

$$\frac{1}{\varepsilon_S(\omega, \boldsymbol{q})} = \frac{1}{\varepsilon_0}\left\{\frac{3(q\lambda)^2 + 8\pi^2}{q\lambda[(q\lambda)^2 + 4\pi^2]} - \frac{32\pi^4}{(q\lambda)^2[(q\lambda)^2 + 4\pi^2]^2} \times \frac{2\varepsilon_+\tilde{\varepsilon}_-(\cosh q\lambda - 1) + \varepsilon_0(\varepsilon_+ + \tilde{\varepsilon}_-)\sinh q\lambda}{\varepsilon_0(\tilde{\varepsilon}_- + \varepsilon_+)\cosh q\lambda + (\varepsilon_0^2 + \tilde{\varepsilon}_-\varepsilon_+)\sinh q\lambda}\right\}. \tag{30}$$

In the particular case of $\varepsilon_0 = \varepsilon_+ = \tilde{\varepsilon}_-$ (30) is consistent with the equation for the form factor $H(q)$ in [13].

In the limit of an extremely thin layer of 2DEG, (29) and (30) yield

$$\varepsilon_S(\omega, \boldsymbol{q}) = \frac{\varepsilon_+ + \tilde{\varepsilon}_-(\omega, \boldsymbol{q})}{2} \equiv \frac{\varepsilon_+}{2} + \frac{\varepsilon_-}{2} \frac{\beta(\omega, \boldsymbol{q}) \tanh ql + 1}{\beta(\omega, \boldsymbol{q}) + \tanh ql}. \tag{31}$$

Further specification of the dielectric function $\varepsilon_S(\omega, \boldsymbol{q})$ requires an explicit form of the function $\beta(\omega, \boldsymbol{q})$, defined by the properties of the system in the $z < -l$ region (cf. (26)). We consider the halfspace $z < -l$ being occupied by a homogeneous 3DEG with a lattice constant ε_- in the region. To find $\beta(\omega, \boldsymbol{q})$ one must solve simultaneously the Poisson equation and kinetic equation for fluctuations $\delta f^{(3)}$ of the 3DEG distribution function (with additional boundary condition on $\delta f^{(3)}$ at $z = -l$). In the case of specular reflection of electrons at the boundary, the problem can be solved easily by an implicit technique using fictitious interface charges [14]. As a result we obtain

$$\beta(\omega, \boldsymbol{q}) = \frac{\varepsilon_- q}{\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}k_z}{k^2[\varepsilon_- + \Delta\varepsilon_{3D}(\omega, \boldsymbol{k})]}; \qquad \boldsymbol{k} = (\boldsymbol{q}, k_z). \tag{32}$$

Here $\Delta\varepsilon_{3D}(\omega, \boldsymbol{k})$ is the contribution of unbounded 3DEG to the dielectric permittivity of the medium (see, e.g., [6]).

The function $\beta(\omega, \boldsymbol{q})$ defined by (32) is widely employed in problems dealing with the interaction between a particle and a semibounded plasma. The calculation of $\beta(\omega, \boldsymbol{q})$ is usually based on the "plasmon pole approximation" [15, 16]. However, in some cases (including that under consideration) the frequencies contributing to the kinetic coefficients may differ much from the plasmon frequency and a problem of more appropriate treatment arises. The calculation of $\beta(\omega, \boldsymbol{q})$ in some limiting cases is given in the Appendix.

5. Relaxation Frequencies of 2DEG, Scattering on 3DEG

For conveniency, in this section we denote values relating to 2DEG (temperature, concentration, effective mass of carriers, etc.), by subscript 2 (e.g. T_2, n_2, m_2) and those of 3DEG – by subscript 3 (T_3, n_3, m_3, etc.).

To calculate the momentum and energy relaxation frequencies of a nondegenerate 2DEG we employ (25) and familiar expressions for 2DEG permittivity [5],

$$\begin{pmatrix} \nu_m \\ \nu_\varepsilon \end{pmatrix} = \frac{e^2 v_{2T}}{2\sqrt{\pi}\, T_2} \int_0^\infty q\, \mathrm{d}q \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \begin{pmatrix} 1 \\ (2\omega/q v_{2T})^2 \end{pmatrix} \mathrm{e}^{-(\omega/q v_{2T})^2} \frac{\operatorname{Im} \varepsilon_S(\omega, q)}{|\varepsilon(\omega, q)|^2}, \tag{33}$$

where $v_{2T} = \sqrt{2T_2/m_2}$ is the thermal velocity of 2DEG.

By using (29) to (31) for $\varepsilon_s(\omega, q)$ we find

$$\frac{\operatorname{Im} \varepsilon_s(\omega, q)}{|\varepsilon(\omega, q)|^2} = \frac{-2\varepsilon_- \operatorname{Im} \beta(\omega, q)\, F(\omega, q)}{\cosh^2 gl\, |\varepsilon_-(1 + \beta \tanh ql) + (\varepsilon_+ + 2\,\Delta\varepsilon_{2D})(\tanh gl)|^2}.$$

In the actual range of ω, q the function $F(\omega, q) \approx 1$, for an extremely thin layer of 2DEG $F(\omega, q) = 1$. For simplicity in what follows we put $F(\omega, q) = 1$.

In the case of low velocities v_{2T} the integration over ω in (33) is cut off by the exponential factor $\exp[-(\omega/qv_{2T})^2]$. Then in the actual range of ω and q we have $\operatorname{Im}\beta \sim -\omega$, $|\beta| \ll 1$ and $\tanh ql \approx 1$. Carring out integration over ω in (33) we arrive at

$$\nu_m \approx \nu_\varepsilon \approx \frac{\varepsilon_- e^2}{m_2} \int_0^\infty \frac{q^2\,dq}{\cosh^2 ql}\left[-\frac{\operatorname{Im}\beta(\omega,q)}{\omega}\right]\frac{1}{\varepsilon^2(q)}. \tag{34}$$

Here $\varepsilon(q) = (1/2)(\varepsilon_+ + \varepsilon_-)(1 + 1/ql_{2sT})$, $l_{2sT} = (\varepsilon_+ + \varepsilon_-)T_2/4\pi e^2 n_2$ is the screening length in the nondegenerate 2DEG, the ratio in square brackets does not depend on ω.

We introduce the screening length in 3DEG as $l_{3s} = v_3/\omega_p$, where v_3 and ω_p are defined in the Appendix.

(i) *The case of* $l \gg l_{3s}$, $v \ll v_3 l/l_{3s}$. The integration region is limited by the value l^{-1}. In this region

$$\operatorname{Im}\beta(\omega,q) \approx -c_3\frac{\omega q v_3}{\omega_p^2}\ln\frac{\omega_p}{qv_3}$$

and (34) implies

$$\nu_m \approx \nu_\varepsilon \approx c_3\frac{e^2\varepsilon_-}{\varepsilon^2(l^{-1})}\frac{l_{3s}^2}{m_2 v_3 l_4}\ln\frac{l}{l_{3s}}.$$

(ii) *The case of* $l \ll l_{3s}$, $v_{2T} \ll v_3$. In the actual region of $l_{3s}^{-1} < q < l^{-1}$ we have $\operatorname{Im}\beta(\omega,q) \approx -c_4\omega\omega_p^2/(qv_3)^2$. Relaxation frequencies are

$$\nu_m \approx \nu_\varepsilon \approx c'\frac{\varepsilon_- e^2}{\varepsilon^2(\tilde q)\,l_{3s}^2 m_2 v_3}.$$

Characteristic wave vector $\tilde q$ and value c' are determined by the relation between relevant characteristic lengths. In the case of

$$q_{\max} \equiv \min(l^{-1}, m_2 v_{2T}/\hbar, m_3 v_3/\hbar, T_2(\varepsilon_- + \varepsilon_+)/2e^2) \gg q_{\min} \equiv \max(l_{3s}^{-1}, l_{2sT}^{-1}, \nu_3/v_3)$$

we have $c' = 2c_4\ln(q_{\max}/q_{\min})$, $\varepsilon(\tilde q) = (\varepsilon_- + \varepsilon_+)/2$. Here ν_3 is the mean collison frequency in 3DEG.

Next we proceed to the case of high velocities v_{2T}, where the exponential factor in (34) is not actual.

(iii) *The case of* $l \gg l_{3s}$, $v_{2T} \gg v_3 l/l_{3s}$. The main contribution to (33) is given by $\omega \approx \omega_p$ and $q \lessapprox l^{-1}$, in this region $\operatorname{Im}\beta(\omega,q) \approx -ql_{3s}$ and we arrive at

$$\begin{pmatrix}\nu_m\\ \nu_\varepsilon\end{pmatrix} \approx \frac{e^2}{\varepsilon^2(l^-)}\frac{v_{2T}l_{3s}\varepsilon_-}{l^3 T_2}\begin{pmatrix}1\\ (v_3 l/v_{2T}l_{3s})^2\end{pmatrix}.$$

(iv) *The case of* $l \ll l_{3s}$, $v_{2T} \gg v_3$. The main contribution to (33) is given by $\omega \approx qv_3$, then $\operatorname{Im}\beta(\omega,q) \approx -(ql_{3s})^{-2}$ and

$$\begin{pmatrix}\nu_m\\ \nu_\varepsilon\end{pmatrix} \approx c''\frac{e^2}{\varepsilon^2(\tilde q)}\frac{v_{2T}\varepsilon_-}{l_{3s}^2 T_2}\begin{pmatrix}1\\ (v_3/v_{2T})^2\end{pmatrix}.$$

Assuming the inequality $q_{\max} \gg q_{\min}$ we get $c'' \approx \ln(q_{\max}/q_{\min})$ and $\varepsilon(\tilde q) = (\varepsilon_- + \varepsilon_+)/2$.

Analysis of the considered limiting cases provides an interpolation form for relaxation frequencies of nondegenerate 2DEG, scattering on the equilibrium 3DEG,

$$\nu_\varepsilon \approx \frac{4\pi e^4 n_3}{\varepsilon^2(\tilde{q})} \frac{(m_2 m_3)^{1/2}}{[T_2 m_3 + W_3 m_2(1 + l^2/l_{3s}^2)]^{3/2}} \frac{1}{1 + l/l_{3s}}, \tag{35}$$

$$\frac{\nu_m}{\nu_\varepsilon} \approx 1 + \frac{m_3}{m_2} \frac{T_2}{W_3(1 + l^2/l_{3s}^2)}. \tag{36}$$

Here the characteristic energy of 3DEG, W_3, is defined in the Appendix, the representative wave vector $\tilde{q}$ is determined by the region of q, giving the main contribution in (33).

It should be observed that in the case of $l \ll l_{3s}$ the frequencies ν_m and ν_ε are essentially the same as for scattering of 3DEG particles on another three-dimensional gas [3, 6]. Relaxation frequencies may logarithmically depend on l, for extremely small l the dependence vanishes.

In the opposite limit ($l \gg l_{3s}(1 + v_{2T}/v_3)$), spatial separation of 2DEG and 3DEG leads to the decrease of the transferred momentum $\hbar q$ (the upper limit is set by $\hbar l^{-1}$). As a result the frequencies ν_m and ν_ε are drawn together even if $m_3 \gg m_2$. Relaxation frequencies depend on l as $\nu_{m,\varepsilon} \sim l^{-4}\varepsilon^{-2}(l^{-1})$, i.e. the frequencies diminish more rapidly than l^{-4}. In the case of $v_{2T} \gg v_3$ in the intermediate region $l_{3s} \ll l \ll \lambda_{3s} v_{2T}/v_3$ we have $\nu_m \sim l^{-3}$ and $\nu_\varepsilon \sim l^{-1}$.

According to (35) and (36), temperature and concentration dependence of $\nu_{m,\varepsilon}$ (if not allowing for the peculiarities in screening) differ from the three-dimensional one only in the case of $l \gg l_{3s}$.

Our consideration is based upon a classical and perturbative treatment of weakly nonideal 2DEG and 3DEG, which imposes the restriction

$$\tilde{q} \ll q_c \equiv \min\,(m_2 v_2/\hbar, m_3 c_3/\hbar, W_2(\varepsilon_+ + \varepsilon_-)/2e^2, W_3\varepsilon_-/e^2)\,.$$

Here $v_2 = \sqrt{2W_2/m_2}$, $W_2 = \max\,(T_2, F_2)$ are 2DEG characteristic velocity and energy, $F_2 = \pi\hbar^2 n_2/m^2$ is the Fermi energy of 2DEG.

Consistent account for quantum and nonperturbative effects provides the automatic cut-off at q_c, i.e. on inverse de Broglie or Landau lengths [3, 6]. For an approximate analytical calculation (within the accuracy of order unity) we can introduce this cut-off artificially. Detailed analysis of all limiting cases leads to the result, valid in the whole range of parameters of weakly nonideal 2DEG and 3DEG,

$$\begin{pmatrix} \nu_\varepsilon \\ \nu_m \end{pmatrix} \approx \frac{e^2}{W_2 v_2 \varepsilon^2(\tilde{q})} \frac{\tilde{\omega}}{\tilde{q} v_3} \left(\frac{\tilde{q} l_{3s}}{1 + \tilde{q}^2 l_{3s}^2}\right)^2 \left(\frac{\tilde{\omega}^2}{2T_2\tilde{q}^2/m_2}\right) \mathscr{L}, \tag{37}$$

where characteristic wave vector and frequency are

$$\tilde{q} = \min\,\{q_{\max}, l_{3s}^{-1} + \eta/l_{2s} + T/\hbar(v_2 + v_3)\}\,; \qquad q_{\max} = \min\,(l^{-1}, q_c)\,,$$

$$\tilde{\omega} = \min\,\{T/\hbar, q_{\max} v_2, (q_{\max} + l_{3s}^{-1} + \eta/l_{2s})\,v_3\}\,,$$

$$\mathscr{L} = 1 + \ln\,\{1 + 1/l_{3s}(q_{\max} + \tilde{\omega}/v_3) + \min\,[q_{\max}, T/\hbar(v_2 + v_3)]/(l_{2s}^{-1} + l_{3s}^{-1})\}\,,$$

$$\varepsilon(q) = (\varepsilon_+ + \varepsilon_-)\,(1 + 1/q l_{2s})/2; \qquad l_{2s} = 2\pi e^2 n_2/W_2\,.$$

The value $\eta = 0$ for ν_ε and $\eta = 1$ for ν_m.

Equation (37) goes over into (35), (36) in the particular case of $T_2 > F_2$ and $q_c > \min\,(l^{-1}, l_{2s}^{-1} + l_{3s}^{-1})$.

Estimations show that in 2DEG momentum relaxation the scattering on 3DEG cannot compete with that on impurities, with the exception of some special cases, where the 3DEG concentration is higher than the impurity one. The case of energy relaxation is more interesting. In the region of temperature near 35 K the frequency of energy relaxation on 3DEG can exceed markedly that on the lattice. In fact, for GaAs–AlGaAs heterostructure ($m_2 \approx m_3 \approx 0.066 m_0$, $\varepsilon_+ \approx \varepsilon_- \approx 12$), for $n_2 = 10^{11}\,\mathrm{cm}^{-2}$, $n_3 = 10^{17}\,\mathrm{cm}^{-3}$, $l = 10\,\mathrm{nm}$ numerical evaluation gives for the energy relaxation on 3DEG the frequency $\nu_\varepsilon \approx 10^9\,\mathrm{s}^{-1}$, whereas the experimental value for scattering on DP and LO phonons is equal to $2 \times 10^8\,\mathrm{s}^{-1}$ [17].

Appendix

To calculate the function $\beta(\omega, q)$ defined by (31) we use the familiar expressions for the dielectric permittivity of a classical, collisionless 3DEG (see, e.g. [6]).

In the limit of $|\omega| \gg kv_3$ we have

$$\varepsilon_- + \Delta\varepsilon_{3D}(\omega, \boldsymbol{k}) \approx \varepsilon_-(1 - \omega_p^2/\omega^2 + i0\,\mathrm{sgn}\,\omega)\,, \tag{A1}$$

in the limit of $|\omega| \ll k_{v_3}$

$$\varepsilon_- + \Delta\varepsilon_{3D}(\omega, \boldsymbol{k}) \approx \varepsilon_-[1 + c_1(\omega_p/kv_3)^2 + ic_2\omega_p^2\omega/(kv_3)^3]\,. \tag{A2}$$

Here $\omega_p^2 = 4\pi e^2 n_3/\varepsilon_- m_3$, $v_3 = \sqrt{2W_3/m_3}$, $W_3 = \max(T_3, F_3)$. Values n_3, T_3, $F_3 = (3\pi^2)^{2/3}\,\hbar^2 n_3^{2/3}/2m_3$, m_3 stand for concentration, temperature, Fermi energy, and effective mass of 3DEG particles. For a nondegenerate 3DEG $c_1 = 2$, $c_2 = 2\sqrt{\pi}$, for a degenerate gas $c_1 = 3$, $c_2 = 3\pi/2$.

We split the function $\beta(\omega, \boldsymbol{q})$ in two parts,

$$\beta(\omega, \boldsymbol{q}) = \beta_{pl}(\omega, \boldsymbol{q}) + \tilde{\beta}(\omega, \boldsymbol{q})\,, \tag{A3}$$

where β_{pl} is the plasmon pole contribution (cf. (A1)), the function $\tilde{\beta}$ is attributed to the contribution of the region $k_z \geqq |\omega|/v_3$ in the integral (31).

In the case of $|\omega| \ll qv_3$ the condition $|\omega| \ll kv_3$ always holds and the asymptotic expression (A2) can be employed. As a result, $\beta_{pl} = 0$ and

$$\tilde{\beta}(\omega, \boldsymbol{q}) = (1 + c_1\omega_p^2/q^2v_3^2)^{-1/2} - i(c_2\omega_p^2\omega/q^3v_3^3)\,\Psi(c_1\omega_p^2/q^2v_3^2)\,, \tag{A4}$$

where

$$\Psi(a) = \frac{1}{a(1+a)}\left[\frac{1+2a}{\sqrt{a(1+a)}}\ln(\sqrt{a} + \sqrt{a+1}) - 1\right].$$

In the case of $|\omega| \gg qv_3$ we have

$$\tilde{\beta}(\omega, \boldsymbol{q}) = \frac{qv_3}{\sqrt{c_1}\,\omega_p}\left(1 - \frac{2}{\pi}\arctan\frac{\omega}{\sqrt{c_1}\,\omega_p}\right) - ic_3\frac{\omega qv_3}{\omega_p^2}\left[\ln\left(1 + \frac{c_1\omega_p^2}{\omega^2}\right) - \frac{1}{1 + \omega^2/c_1\omega_p^2}\right], \tag{A5}$$

$$\beta_{pl}(\omega, \boldsymbol{q}) = \omega^2/(\omega - \omega_p + i0)(\omega + \omega_p + i0)\,. \tag{A6}$$

In three regions of the (ω, qv_3) plane (A4), (A5) for $\tilde{\beta}$ simplify for $|\omega|$, $qv_3 \ll \omega_p$ to

$$\tilde{\beta}(\omega, \boldsymbol{q}) \approx qv_3/\sqrt{c_1}\,\omega_p - ic_3(\omega qv_3/\omega_p^2) \ln[(\omega_p/(\omega + qv_3)], \tag{A7}$$

for $\omega_p \ll |\omega| \ll qv_3$ to

$$\tilde{\beta}(\omega, \boldsymbol{q}) \approx 1 - ic_4\omega\omega_p^2/(qv_3)^3, \tag{A8}$$

for $\omega_p \ll qv_3 \ll |\omega|$ to

$$\tilde{\beta}(\omega, \boldsymbol{q}) \approx 2qv_3/\pi\omega - ic_5qv_3\omega_p^2/\omega^3. \tag{A9}$$

In the nondegenerate case $c_3 = 1/2\sqrt{\pi}$, $c_4 = 8/3\sqrt{\pi}$, $c_5 = 1/\sqrt{\pi}$, in the degenerate case $c_3 = 1/6$, $c_4 = 2$, $c_5 = 3/4$.

With the help of (A3), (A6) to (A9) one can construct an interpolation for all frequency and wave vector values,

$$\beta(\omega, \boldsymbol{q}) \approx \frac{\omega^2\vartheta(|\omega| - qv_3)}{(\omega - \omega_p + i0)(\omega + \omega_p + i0)} + \frac{qv_3}{\sqrt{c_1}\,\omega_p + \pi\omega/2 + qv_3} - i\frac{qv_3\omega_p^2\omega\{1 + \ln[1 + \omega_p/(\omega + qv_3)]\}}{\omega_p^4/c_3 + (qv_3)^4/c_4 + \omega^4/c_5}.$$

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