

EFFECTS OF DIELECTRIC MISMATCH ON THE PLASMONS IN QUANTUM WELL SYSTEMS

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In the framework of random-phase approximation theory and applying the self-consistent field method, we study the properties of collective charge density excitations in single quantum well in consideration of dielectric mismatch. We analytically give the general solution of the image potential to the one dimensional Green's function. Our numerical results demonstrate that the dielectric mismatch between the barrier and well materials significantly changes the frequency of the intra- and inter-subband plasmon modes in contrast with dielectric match. We reasonably conclude that the image potential affects the intra- and inter-subband plasmon modes in a different way.

Keywords: Collective excitations; dielectric mismatch; dielectric response function.

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1. Introduction

For many years a large amount of works have been done on the properties of the elementary collective excitations (plasmons) in two-dimensional electron gas (2DEG) system such as quantum wells and superlattices in both theory and experiment. That is due, not only to the technological importance in microelectronic and optoelectronic devices but also the fact that the study of quasi-2DEG has revealed remarkable phenomena associated with electron–electron interactions under reduced dimensions. For the collective transitions of quasi-2DEG, the intrasubband plasmon goes as $q^{1/2}$ in the long wavelength limit in two dimensions, and its dispersion is influenced by the finite thickness and by coupling to higher subbands.¹ The intersubband transitions of electrons correspond to charge-density oscillations perpendicular to the plane, and there exists a collective mode associated with them called the intersubband plasmon.^{2,3} The intra- and inter-subband plasmon modes

in single quantum well (SQW) systems have been extensively studied in the case of dielectric match, which is based on the argument that the dielectric mismatch effect brings only little influence on the electronic ground-state properties of the system^{4,5} because of the small difference of the dielectric constant in materials constituting the actual heterostructure. However, some authors have investigated the image potential in semiconductor interfaces and heterostructures, and find that the image potential is important.^{6–10} Theoretically, the question that how much effect the image potential will bring on the collective excitations of quasi-2DEG is left unknown and it is worthy to be confirmed. Moreover, if the dielectric constants of two materials constituting heterostructure are quite different, what results can be expected when taking into account of the dielectric mismatch? This is just the issue of this paper.

In this paper, we survey the properties of the plasmons in SQW in consideration of the image potential due to dielectric mismatch inside and outside the quantum well. In random-phase approximation (RPA) and applying self-consistent field (SCF) theory, we present the response function of quasi-2DEG. Also, we derive the general solution of the image potential from the Green's function. Such general solution is much more convenient than the series expression shown by Eq. (10) in Ref. 10, where the authors ignored the dependence of the image potential on the transverse coordinate (x, y) . We carry out numerical calculations of the dispersion spectrum of the collective excitation modes for different structural parameters. We find that the dispersion spectrum of the plasmon modes strongly depends on the mismatch degree of the dielectric constants inside and outside the quantum well. It is concluded that the image potential plays an important role in determining the behaviors of intra- and inter-subband plasmon modes of 2DEG when considering dielectric mismatch.

The rest parts of the paper are organized as follows. In Section 2, we present the basic formulae for the calculations of the density response function and the detailed expressions of the image potential arising from dielectric mismatch between adjacent layers. In Section 3, we report the numerical results with discussion. Finally, a brief summary of our main findings is given in Section 4.

2. Model and Formulas

For a quasi-2DEG system, the wave function of single particle, $\Psi_{l,\mathbf{k}}(\rho, z)$, can be factorized into an in-plane part $\exp(i\mathbf{k}\rho)$ and a subband function $\psi_l(z)$, which is independent of the in-plane wavevector $\mathbf{k}$. Here z denotes the direction perpendicular to the QW and $\rho = (x, y)$ represents the in-plane position. According to the effective mass theory, the 1D Schrödinger equation governing the single electron motion can be written as

$$\left[-\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m(z)} \frac{d}{dz} + V_{\text{eff}}(z) \right] \psi_l(z) = E_l \psi_l(z), \quad (1)$$

where $m(z)$ is the effective mass of electrons and it depends on the spatial position. Here, $V_{\text{eff}}(z) = V_0(z) + V_H(z)$ is an effective potential, which is composed of a bare potential $V_0(z)$ arising from the discontinuity of the conduction band of electrons between the barrier and well materials, and a Hartree potential $V_H(z)$ due to the electrostatic potential of the mobile electrons of quasi-2DEG along with the spatial fixed ionized donors. Here we choose the growth direction of heterostructure to be along the z -axis. The Hartree potential $V_H(z)$ is determined by the Poisson equation

$$\frac{d}{dz}\epsilon(z)\frac{d}{dz}V_H(z) = -\frac{e^2}{\epsilon_0}[n(z) - N_d(z)]. \quad (2)$$

Here, ϵ_0 is the permittivity in vacuum, $\epsilon(z)$ represents the dielectric constant dependent on the spatial position, and $N_d(z)$ is the donor density. The electronic density along the z direction is given by

$$n(z) = \sum_l |\psi_l(z)|^2 n_l \quad (3)$$

with an areal density in the l th electronic subband as

$$n_l = \frac{m_w^*}{\pi\hbar^2}(E_F - E_l)\Theta(E_F - E_l), \quad (4)$$

where m_w^* represents the effective electron mass in the quantum well and $\Theta(x)$ the Heaviside unit step function. It is noted that the exchange-correlation potential can be neglected now owing to its small influence on the electron collective excitations under usual electron density.

We employ an iteration technique to solve the above Schrödinger equation with the effective potential self-consistently. Applying the “turning point” technique to the Numerov algorithm,^{11,12} the Schrödinger equation can be solved by using the boundary conditions $\psi_l(\pm\infty) = 0$. The total areal density n_T , the electronic density in the z direction $n(z)$, the depletion lengths d_1 and d_2 , and the Hartree potential $V_H(z)$ are obtained, not given as an initial conditions.¹³ With these obtained functions and parameters, we repeat the above to solve the Schrödinger equation and obtain the further solutions in the next loop till the precision of the convergent solution is satisfied. Finally, the SCF wavefunctions of the single particles, $|\alpha\rangle, |\beta\rangle \dots$, can be obtained.

We now focus on the density response of a quasi-2DEG for an external, harmonic longitudinal electric field using SCF method and linear response theory. In the RPA, the induced potential has the form

$$V^{\text{in}}(\mathbf{r}) = \int U(\mathbf{r}, \mathbf{r}') \delta n(\mathbf{r}') d\mathbf{r}', \quad (5)$$

where $\delta n(\mathbf{r})$ denotes the fluctuation of the density of electrons and it can be evaluated according to the standard linear response theory as

$$\delta n(\mathbf{r}) = \sum_{\alpha\beta} \Psi_\alpha^*(\mathbf{r}) \Psi_\beta(\mathbf{r}) \Pi_{\alpha\beta} \langle \beta | V^T(\mathbf{r}) | \alpha \rangle \quad (6)$$

with the generalized polarizability

$$\Pi_{\alpha\beta} = \frac{f_{\alpha} - f_{\beta}}{E_{\alpha} - E_{\beta} + (\hbar\omega + i\eta)}.$$

We expand the matrix element of Eq. (5) as

$$\begin{aligned} V_{\alpha\beta}^{\text{in}}(\mathbf{r}) &= \langle \alpha | V^{\text{in}}(\mathbf{r}) | \beta \rangle \\ &= \sum_{\mu\nu} U_{\alpha\beta,\mu\nu}(\mathbf{r}, \mathbf{r}') \Pi_{\mu\nu} V_{\nu\mu}^T(\mathbf{r}), \end{aligned} \quad (7)$$

where the Coulomb matrix element is

$$U_{\alpha\beta,\mu\nu}(\mathbf{r}, \mathbf{r}') = \sum_{\mu\nu} \iint d\mathbf{r} d\mathbf{r}' \Psi_{\alpha}^*(\mathbf{r}) \Psi_{\beta}^*(\mathbf{r}') U(\mathbf{r}, \mathbf{r}') \Psi_{\mu}^*(\mathbf{r}') \Psi_{\nu}^*(\mathbf{r}). \quad (8)$$

By using the relation $V_{\alpha\beta}^{\text{ex}}(\mathbf{r}) = V_{\alpha\beta}^T - V_{\alpha\beta}^{\text{in}}$, we can express the external potential in terms of the screened one as

$$V_{\alpha\beta}^{\text{ex}}(\mathbf{r}) = \sum_{\mu\nu} [\delta_{\alpha\nu} \delta_{\beta\mu} - U_{\alpha\beta,\mu\nu}(\mathbf{r}, \mathbf{r}') \Pi_{\mu\nu}] V_{\nu\mu}^T(\mathbf{r}), \quad (9)$$

which defines the response function

$$\overset{\leftrightarrow}{\varepsilon}_{\alpha\beta\delta_{\mu\nu}} = \delta_{\alpha\nu} \delta_{\beta\mu} - U_{\alpha\beta,\mu\nu}(\mathbf{r}, \mathbf{r}') \Pi_{\mu\nu}. \quad (10)$$

The condition for plasmon modes is the vanishing of the determinant of the dielectric tensor,

$$\det[\overset{\leftrightarrow}{\varepsilon}_{\alpha\beta,\mu\nu}] = 0.$$

Considering the above-addressed forms of the wavefunctions in the QW, we have

$$\begin{aligned} |\alpha\rangle &= |l, \mathbf{q}\rangle = \psi_l(z) e^{i\mathbf{q}\cdot\rho}, \\ |\beta\rangle &= |l', \mathbf{q}'\rangle = \psi_{l'}(z) e^{i\mathbf{q}'\cdot\rho}, \\ |\mu\rangle &= |n, \mathbf{k}\rangle = \psi_n(z) e^{i\mathbf{k}\cdot\rho}, \\ |\nu\rangle &= |n', \mathbf{k}'\rangle = \psi_{n'}(z) e^{i\mathbf{k}'\cdot\rho}. \end{aligned} \quad (11)$$

Substituting the above expressions into Eq. (10), the dielectric function in the plane-wave Fourier representation can be expressed as¹³

$$\varepsilon_{ll',nn'}(\mathbf{q}, \omega) = \delta_{ln'} \delta_{l'n} - \Pi_{nn'}(\mathbf{q}, \omega) U_{ll',nn'}(\mathbf{q}, z, z') \quad (12)$$

with the polarizability function

$$\Pi_{nn'}(\mathbf{q}, \omega) = 2 \sum_{\mathbf{k}} \frac{f_n(\mathbf{k}) - f_{n'}(\mathbf{k} + \mathbf{q})}{E_n(\mathbf{k}) - E_{n'}(\mathbf{k} + \mathbf{q}) + \hbar\omega}$$

and

$$U_{ll',nn'}(\mathbf{q}, z, z') = \iint dz dz' \psi_l^*(z) \psi_{l'}(z) U(\mathbf{q}, z, z') \psi_n^*(z') \psi_{n'}(z').$$

Here, $\psi_l(z)$ is the wavefunction of the single electron for the l th subband due to the quantum confinement along the z axis. Here $E_l(\mathbf{k}) = E_l(0) + \hbar^2 k^2 / 2m_w^*$ ($k = |\mathbf{k}|$) is the single-electron energy in the l th subband. $f_n(\mathbf{k})$ is the Fermi-Dirac occupation probability of electrons at the state $|n, \mathbf{k}\rangle$, with the value of unity or zero at zero temperature. $U(\mathbf{q}, z, z')$ is the plane-wave Fourier transform spectrum of the Coulomb interaction term $U(\mathbf{r}, \mathbf{r}')$. The dispersion relation of the collective excitations is determined by the zero of the determinant of the dielectric function given by Eq. (12): $\det[\varepsilon_{ll', nn'}(\mathbf{q})] = 0$.

It is well known that when neglecting the difference of the static dielectric constants inside and outside the quantum well, the Fourier expansion in Eq. (12) has a form of

$$U(\mathbf{q}, z, z') = \frac{e^2}{2\epsilon_r \epsilon_0 q} e^{-q|z-z'|},$$

with $q = |\mathbf{q}|$. In the previous studies, this difference is omitted as a minor factor dominating the collective excitations, assuming its contribution less than 10 percentage points.³ Now we are interested in calculating analytically the image potential due to dielectric mismatch. Supposing an electron charge is located at $\mathbf{r}'(\rho', z')$, the image potential is governed by the Poisson equation as

$$\nabla^2 U(\mathbf{r}, \mathbf{r}') = -\frac{e}{\epsilon_0 \epsilon(z)} \delta(\rho - \rho') \delta(z - z') \quad (13)$$

with

$$\epsilon(z) = \begin{cases} \epsilon_1 & z < -a/2 & \text{Region } L \text{ (left barrier)} \\ \epsilon_2 & |z| \leq a/2 & \text{Region } W \text{ (quantum well)} \\ \epsilon_3 & z > a/2 & \text{Region } R \text{ (right barrier)} \end{cases}.$$

Substituting the Fourier expansion of $U(\mathbf{r}, \mathbf{r}')$ and $\delta(\rho, \rho')$ into Eq. (13), we derive the equation of Fourier component of the image potential to be satisfied

$$\frac{d^2 U(\mathbf{q}, z, z')}{dz^2} - q^2 U(\mathbf{q}, z, z') = -\frac{e}{\epsilon(z)} \delta(z - z'). \quad (14)$$

Equation (14) is a standard one dimensional Green function equation, and its general solution may be expressed as follows:

- (i) $z' > a/2$ (the electron is located at the right barrier R), Eq. (14) has the solutions as

$$U_R(\mathbf{q}, z, z') = \frac{e^2}{2\epsilon_0 \epsilon_3 q} \left[\frac{(\epsilon_3 + \epsilon_2)(\epsilon_2 - \epsilon_1)e^{-q(z+z')} + (\epsilon_3 - \epsilon_2)(\epsilon_2 + \epsilon_1)e^{-q(z+z'-2a)}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} + e^{-q|z-z'|} \right], \quad (15)$$

$$U_W(\mathbf{q}, z, z') = \frac{e^2}{\epsilon_0 q} \left[\frac{(\epsilon_2 - \epsilon_1)e^{-q(z+z')} + (\epsilon_2 + \epsilon_1)e^{q(z-z'+a)}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right], \quad (16)$$

and

$$U_L(\mathbf{q}, z, z') = \frac{2\epsilon_2 e^2}{\epsilon_0 q} \left[\frac{e^{q(z-z'+a)}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right] \quad (17)$$

for regions R , W and L , respectively.

- (ii) $-a/2 \leq z' \leq a/2$ (the electron is in the well), it is straightforward from Eq. (14) to get

$$U_R(\mathbf{q}, z, z') = \frac{e^2}{\epsilon_0 q} \left[\frac{(\epsilon_2 - \epsilon_1)e^{-q(z+z')} + (\epsilon_2 + \epsilon_1)e^{-q(z-z'-a)}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right], \quad (18)$$

$$\begin{aligned} U_W(\mathbf{q}, z, z') = \frac{e^2}{2\epsilon_0 \epsilon_2 q} & \left\{ \frac{(\epsilon_2 - \epsilon_1)(\epsilon_2 - \epsilon_3)[e^{-q(z-z'+a)} + e^{q(z-z'-a)}]}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right. \\ & + \frac{(\epsilon_2 - \epsilon_1)(\epsilon_2 + \epsilon_3)e^{-q(z+z')} + (\epsilon_2 + \epsilon_1)(\epsilon_2 - \epsilon_3)e^{q(z+z')}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \\ & \left. + e^{-q|z-z'|} \right\}, \quad (19) \end{aligned}$$

and

$$U_L(\mathbf{q}, z, z') = \frac{e^2}{\epsilon_0 q} \left[\frac{(\epsilon_2 + \epsilon_3)e^{q(z-z'+a)} + (\epsilon_2 - \epsilon_3)e^{q(z+z')}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right] \quad (20)$$

for regions R , W and L , respectively.

- (iii) $z' < -a/2$ (the electron is in the left barrier), the image potential has the forms as

$$U_R(\mathbf{q}, z, z') = \frac{2\epsilon_2 e^2}{\epsilon_0 q} \left[\frac{e^{-q(z-z'-a)}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right], \quad (21)$$

$$U_W(\mathbf{q}, z, z') = \frac{e^2}{\epsilon_0 q} \left[\frac{(\epsilon_2 - \epsilon_3)e^{q(z+z')} + (\epsilon_2 + \epsilon_3)e^{-q(z-z'-a)}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right], \quad (22)$$

and

$$\begin{aligned} U_L(\mathbf{q}, z, z') = \frac{e^2}{2\epsilon_0 \epsilon_1 q} & \left[\frac{(\epsilon_1 + \epsilon_2)(\epsilon_2 - \epsilon_3)e^{q(z+z')} + (\epsilon_1 - \epsilon_2)(\epsilon_2 + \epsilon_3)e^{q(z+z'+2a)}}{(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3)e^{qa} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3)e^{-qa}} \right. \\ & \left. + e^{-q|z-z'|} \right] \quad (23) \end{aligned}$$

for three different regions R , W and L .

Till now, the analytical forms of the image potential are fully acquired. It is worth pointing out that the solution of Eq. (13) is also expressed into a series sum from a finite number of images, where each has different position and charge, as shown by Eq. (10) in Ref. 10. The authors¹⁰ ignored the dependence of the image potential on the transverse coordinate (x, y). Obviously for the purpose in this paper, the general solution of Eq. (14) is much more convenient than the series expression of Eq. (13). For the case of the symmetric SQW, we have $\epsilon_1 = \epsilon_3$. Substituting the derived expressions of the image potential of Eqs. (15)–(23) into Eq. (12), the collective excitations in the case of dielectric mismatch are totally decided theoretically.

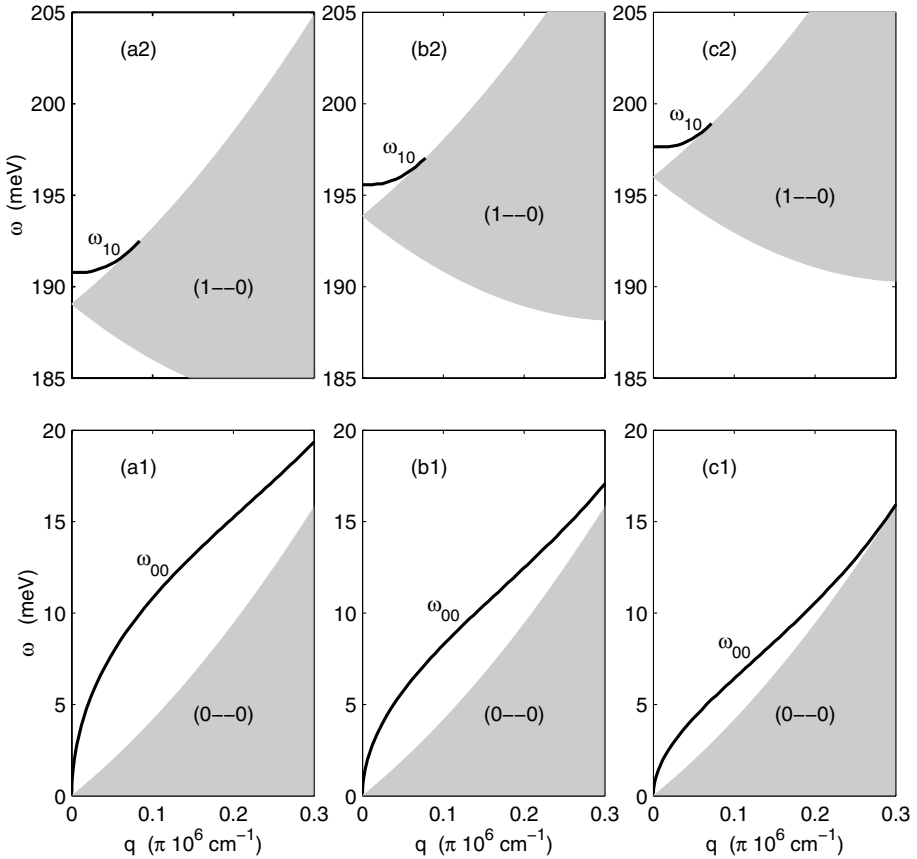


Fig. 1. Dispersion spectra of the plasmon modes in a SQW structure for different ratios of the dielectric constant ϵ_1 of the barrier layer to that ϵ_2 of the well layer: (a1)–(a2) for $\epsilon_1/\epsilon_2 = 0.5$, (b1)–(b2) for $\epsilon_1/\epsilon_2 = 1.0$, and (c1)–(c2) for $\epsilon_1/\epsilon_2 = 2.0$. The barrier height is 300 meV and the well width $a = 5$ nm. The electron density and the donor density used here are $1.6 \times 10^{11} \text{ cm}^{-2}$, $4.0 \times 10^{17} \text{ cm}^{-3}$, respectively.

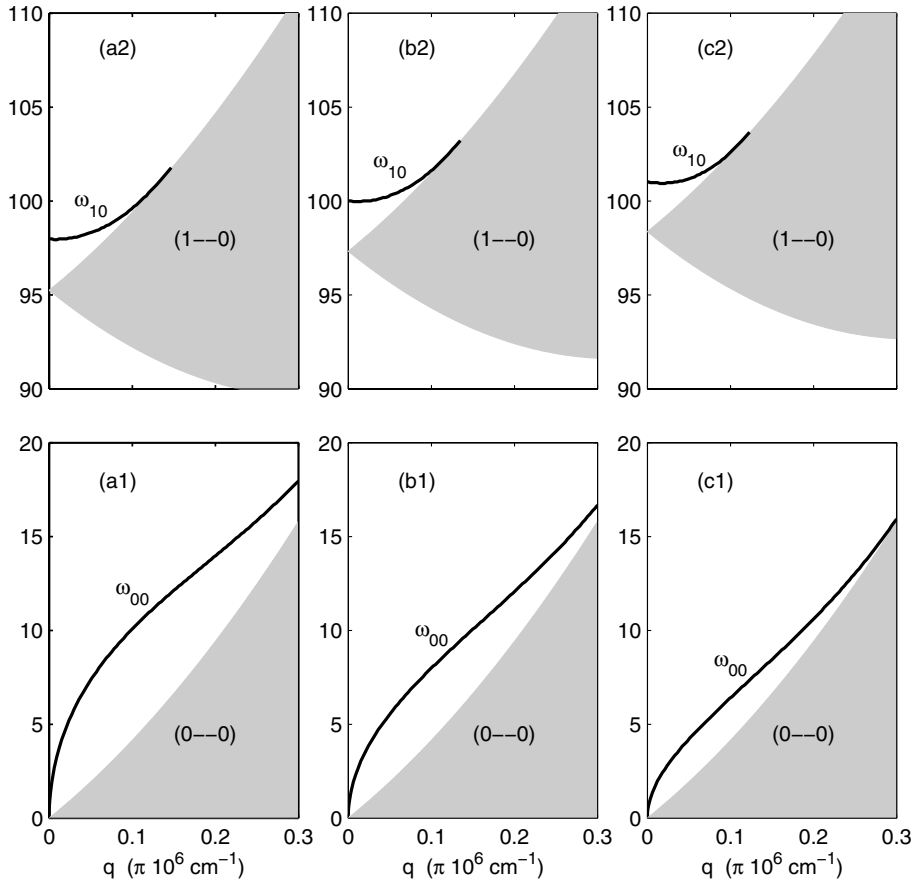


Fig. 2. Same as Fig. 1 except for a broader well of $a = 10$ nm: (a1)–(a2) correspond to $\epsilon_1/\epsilon_2 = 0.5$, (b1)–(b2) to $\epsilon_1/\epsilon_2 = 1.0$, and (c1)–(c2) to $\epsilon_1/\epsilon_2 = 2.0$.

3. Numerical Results and Discussion

We show the plasmon dispersion spectrum of 2DEG in a SQW with dielectric mismatch in detail in Figs. 1 and 2 for different well widths. In the following numerical calculations, we assume the same electron mass inside and outside the quantum well as $m_w^* = 0.067m_0$ with m_0 being the free electron mass for simplicity. The typical electron density and the donor density in this system are chosen to be $1.6 \times 10^{11} \text{ cm}^{-2}$ and $4.0 \times 10^{17} \text{ cm}^{-3}$, respectively.

Figure 1 displays the dispersion spectra of the single particle excitations and the plasmons in a SQW with the barrier height of 300 meV and the well width of $a = 5$ nm: (a1)–(a2) for the dielectric ratio $\epsilon_1/\epsilon_2 = 0.5$, (b1)–(b2) for $\epsilon_1/\epsilon_2 = 1.0$, and (c1)–(c2) for $\epsilon_1/\epsilon_2 = 2.0$. The ratio ϵ_1/ϵ_2 represents the dielectric mismatch degree between the barrier and the well materials in a SQW. Here, the relative dielectric constant inside the quantum well ϵ_2 remains unchanged as $\epsilon_2 = 12.8$.

ω_{00} and ω_{10} branches denote the intrasubband and intersubband modes of the collective excitations of quasi-2DEG, and the shadow areas stand for the single particle excitation (SPE) continua. It is evident that all the (0-0) SPE continua almost remain unchanged for different dielectric mismatches, however, for the (1-0) SPE continuum, the larger the value of ϵ_1/ϵ_2 is, the upper the (1-0) SPE continuum moves remarkably. In the long wavelength limit $q \rightarrow 0$, the mode ω_{00} approaches zero. Compared Fig. 1(a1) with (b1) and (c1), we observe that the rising speed of the mode ω_{00} with q is adversely reduced for the greater value of the ratio ϵ_1/ϵ_2 . As for the intersubband excitation ω_{10} , the case is quite different: The frequency of the mode ω_{10} at $q = 0$ is enhanced upper for the larger ϵ_1/ϵ_2 , as shown in Figs. 1(a2), (b2), and (c2), for instance, its value at $q = 0$ is increased from 190.8 to 195.6, and to 197.7 meV for $\epsilon_1/\epsilon_2 = 0.5, 1.0$ and 2.0 , respectively. As q increases, the mode ω_{10} rises up monotonically and finally merges into its corresponding (1-0) SPE continuum. Owing to the lowering of the mode ω_{00} and the ascension of its corresponding mode ω_{10} for the greater dielectric ratio ϵ_1/ϵ_2 , the spacing between the intrasubband plasmon mode and the intersubband plasmon mode is increased substantially.

The above obtained results can be interpreted from a simple argument as follows: For a charge Q placed near an interface between two different dielectrics, the image potential is attractive for charges on the low-dielectric-constant side of the interface and repulsive for charges on the high-dielectric-constant side of the interface.¹⁴ Thus, it can be concluded that when the dielectric constant ϵ_1 outside the quantum well is less than that ϵ_2 inside the quantum well, i.e. in the case of $\epsilon_1/\epsilon_2 = 0.5$, the image potential for electrons inside the well is repulsive, which leads to the significant enhancement of the interaction between electrons inside the well. As a result, the frequency of the intrasubband plasmon associated with the collective oscillation of electrons should be increased. On the contrary, when the dielectric constant ϵ_1 of the barrier layer is greater than that ϵ_2 of the well, i.e. in the case of $\epsilon_1/\epsilon_2 = 2.0$, the attractive image potential for electrons inside the well tends to weaken the interaction of electrons inside the well, which results in the lowering of the frequency of the intrasubband plasmon mode. As for the higher frequency of the mode ω_{10} in the case of $\epsilon_1/\epsilon_2 = 2.0$ in contrast with other two cases, it belongs to the fact that, when the dielectric constant ϵ_1 outside the quantum well is much greater than that inside the well, the effective potential of the SQW is essentially enhanced, so the spacing of energy levels between the ground state and the first excited state of the electrons inside the well is broadened. As a result, the spacing between the modes ω_{10} and ω_{00} is also increased for the greater dielectric ratio ϵ_1/ϵ_2 .

Figure 2 depicts the variation of the dispersion spectra of the plasmons for a larger width of the well, $a = 10$ nm, with different dielectric contrasts: (a1)–(a2) correspond to $\epsilon_1/\epsilon_2 = 0.5$, (b1)–(b2) to $\epsilon_1/\epsilon_2 = 1.0$, and (c1)–(c2) to $\epsilon_1/\epsilon_2 = 2.0$, respectively. Here, other parameters are chosen the same as those in Fig. 1. Similar phenomena can be observed now for the variation of the dielectric ratio ϵ_1/ϵ_2 .

Comparing Fig. 1 with Fig. 2, it is clearly seen that the (0-0) SPE continuum remains almost unchanged for a broader well, while the frequency of the intrasubband plasmon mode ω_{00} is lowered a little. For the width-increased quantum well, we find that the (1-0) SPE continuum falls down substantially, for example, its value at $q = 0$ is decreased from 189.1 (193.9, 196) meV to 95.3 (97.3, 98.4) meV for $\epsilon_1/\epsilon_2 = 0.5$ (1.0, 2.0), respectively, when comparing Fig. 1 with Fig. 2. Moreover, the intersubband plasmon branch ω_{10} is lowered accordingly, for instance, from 190.8 (195.6, 197.7) meV to 97.9 (100, 101) meV for $\epsilon_1/\epsilon_2 = 0.5$ (1.0, 2.0), respectively, at $q = 0$. Consequently, the separation between the modes ω_{10} and ω_{00} is narrowed for a broader well, as expected. We also observe that the span of the mode ω_{10} in wavenumber range is extended significantly for a wider quantum well, comparing Figs. 1(a2), (b2), and (c2) with Figs. 2(a2), (b2), and (c2). When lowering the barrier height, the effects are analogous to the broadening of the well.

4. Summary

We have investigated the properties of the plasmons in SQW in consideration of the image potential due to the dielectric mismatch inside and outside the quantum well. We present the analytical expressions of the image potential, which is a general solution to the standard one dimensional Green's function and is much more convenient than the series expression of Eq. (13). We numerically display the dispersion spectra of the collective excitation modes for various physical parameters. Our results show that the frequency of the plasmon modes and SPE continua is strongly related to the mismatch degree of dielectric constants inside and outside the quantum well. When the dielectric constant of the barrier material is less than that of the well, the repulsive image potential of electrons inside the well strengthens the interaction between electrons inside the well and leads to the greater rising speed of the intrasubband plasmon modes ω_{00} with q , when comparing with the case of dielectric match. Moreover, the frequency of the intersubband modes falls down, consequently, the spacing between the intra- and inter-subband modes is narrowed accordingly. On the contrary, when the dielectric constant of the barrier is greater than that of the well material, the situation becomes reverse fully. The frequency of corresponding intrasubband plasmons ω_{00} is lowered remarkably, which stems from the attractive image potential of electrons inside the well now. However, the intersubband mode rises adversely due to the expansion of the separation of the energy levels between the ground state and the first excited state of electrons inside the well. We reasonably attribute these results to the action of the image potential.

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