S. A. Klein

J. C. Theilacker

Solar Energy Laboratory
University of Wisconsin-Madison
Madison, Wis. 53706

An Algorithm for Calculating Monthly-Average Radiation on Inclined Surfaces

An analytical method is developed for estimating $\bar{R}$, the ratio of the long-term monthly-average daily radiation on an inclined surface to that on a horizontal surface. This method differs from the Liu and Jordan [1] method in the manner in which the beam radiation component is determined. The method is applicable for surfaces of any orientation. $\bar{R}$ values calculated in this manner and by the Liu and Jordan method are compared with integrated hourly calculations for 23 years in Madison, WI, Albuquerque, NM and Miami, FL; the comparisons show that the method described in this paper agrees more closely with the integrated hourly calculations, especially for surfaces facing east or west of south.

1 Introduction

A method of estimating the monthly-average[1] solar radiation incident on surfaces oriented directly towards the equator has been developed by Liu and Jordan [1]. The method was later extended for surfaces facing east or west of south by Klein [2]. In this method, $\bar{H}_T$, the monthly-average daily total radiation incident on an inclined surface, is related to $\bar{H}$, the monthly-average daily radiation on a horizontal surface, by a factor $\bar{R}$.

$$\bar{H}_T = \bar{R}\bar{H} \tag{1}$$

$\bar{R}$ is estimated as the sum of three terms corresponding to the beam, diffuse, and ground-reflected radiation incident on the inclined surface. Assuming diffuse and ground-reflected radiation to be isotropic, the Liu and Jordan [1] relationship for $\bar{R}$ can be written:

$$\bar{R} = \left[1 - \left(\frac{\bar{H}_d}{\bar{H}}\right)\right]\bar{R}_b + \left(\frac{\bar{H}_d}{\bar{H}}\right)\left(\frac{1+\cos\beta}{2}\right) + \rho\left(\frac{1-\cos\beta}{2}\right) \tag{2}$$

where

- β is the tilt of the surface from horizontal
- ρ is the ground reflectance
- $(\bar{H}_d/\bar{H})$ is the monthly-average diffuse radiation fraction
- $\bar{R}_b$ is the ratio of the average beam radiation on the inclined surface to that on a horizontal surface

In general, $\bar{R}_b$ is a function of the transmittance of the atmosphere. However, Liu and Jordan suggest that $\bar{R}_b$ can be approximated to be the ratio of the monthly extraterrestrial radiation on the inclined surface to that on a horizontal surface. This approximation results in the analytical expressions for $\bar{R}_b$ given in equation (8) (for surfaces facing directly towards the equator) and equation (11) (for surfaces facing east or west of south) of reference [2]. Liu and Jordan indicate that these expressions for $\bar{R}_b$ are correct only for south-facing surfaces during periods of solar equinox (i.e., March and September). As shown in section 3, this manner of estimating $\bar{R}_b$ can result in significant errors in the calculated values of $\bar{R}$, particularly for surfaces facing east or west of south.

The isotropic assumption for the diffuse component generally results in a conservative (i.e., low) estimate of the monthly-average radiation on surfaces oriented directly towards the equator; for surfaces oriented east or west of south, the isotropic assumption may result in an overestimate of the radiation on an inclined surface. The validity of the isotropic assumption has been questioned by Hay [3]. Hay has developed a model which, like the Liu and Jordan model, separately considers the beam, diffuse, and ground-reflected radiation components. In Hay's model however, a portion of the diffuse component is treated as circumsolar, rather than as isotropic. Ground-reflected radiation is assumed to be isotropic. Although formulated somewhat differently, Hay, like Liu and Jordan, calculates the ratio of beam radiation on an inclined surface to that on a horizontal surface from geometrical considerations.

An alternate means of estimating the beam component was proposed by Page [4]. Page developed a table of values of $\bar{R}_b$ for five surface orientations and a range of latitudes by integrating the instantaneous direct radiation incident on inclined and horizontal surfaces over an average day. The instantaneous direct radiation intensities were obtained from a standard radiation curve representative of tropical conditions. A comparison of values of $\bar{R}_b$ obtained using the Liu and Jordan and Page methods appears in reference [2]. A

[1] Monthly-average refers to the average conditions which occur for a particular month over a long term, such as ten or more years.

Contributed by the Solar Energy Division for publication in the JOURNAL OF SOLAR ENERGY ENGINEERING. Manuscript received by the Solar Energy Division May 12, 1980; revised manuscript received October 2, 1980.

disadvantage of Page's method (aside from its dependence on a tropical reference climate) is that information is given in tabular, rather than analytical form.

Collares-Pereira and Rabl [5, 6] describe an analytical method of estimating the monthly-average daily radiation incident on an inclined surface between specified times of day. This method can be applied to determine $\bar{R}$ if the specified times are sunrise and sunset. However, the equations given in reference [6] are not directly applicable for this purpose since the sunrise to sunset period for the beam component of radiation on an inclined surface may be different than that for the diffuse or ground-reflected component. Also, expressions for the time of sunrise and sunset on a surface facing east or west of south are not given.

In this paper, a new algorithm for $\bar{R}$ is formulated for any surface orientation. Values of $\bar{R}$ calculated from this algorithm are compared with results obtained from Liu and Jordan method (equation (2)), and with results of hourly calculations using 23 years of actual radiation data in each of three locations.

2 Algorithm for $\bar{R}$

$\bar{R}$, the monthly-average ratio of total radiation on an inclined surface to that on a horizontal surface, can be found by integrating I_T, the instantaneous (often approximated with hourly) radiation on an inclined surface, and I, the instantaneous radiation on a horizontal surface over a period of N days. (N should be at least 300 days, corresponding to ten years of data for each month, in order to accurately represent long-term average conditions.)

$$\bar{R} = \frac{\sum_{\text{Day}=1}^{N} \int_{t_{SR}}^{t_{SS}} I_T dt}{\sum_{\text{Day}=1}^{N} \int_{t_{SR}}^{t_{SS}} I\, dt} \tag{3}$$

It should be noted that t_{SR} and t_{SS}, the sunrise and sunset times, will in general differ for horizontal and inclined surfaces. Further, the sunrise and sunset times for beam radiation on an inclined surface may be different than the corresponding values for the diffuse and ground-reflected components.

If measurements of I are available, I_T can be estimated by separately considering the beam, diffuse, and ground-reflected radiation components. Assuming diffuse and ground-reflected radiation to be isotropic, I_T can be expressed:

$$I_T = (I - I_d) R_b + I_d \left(\frac{1+\cos\beta}{2}\right) + \rho I \left(\frac{1-\cos\beta}{2}\right) \tag{4}$$

where I_d is the instantaneous diffuse radiation on a horizontal surface and R_b is the ratio of instantaneous beam radiation on the inclined surface to that on a horizontal surface. As shown in Duffie and Beckman [7], R_b is a geometrical variable which is equal to the ratio of the cosine of the solar incidence angle (on the inclined surface) to the cosine of the solar zenith angle.

$$R_b = \frac{\cos\theta_i}{\cos\theta_z} = \frac{A\cos\omega - B + C\sin\omega}{\cos\omega - \cos\omega_s} \tag{5}$$

where

$$A = \cos\beta + \tan\phi \cos\gamma \sin\beta \tag{6}$$

$$B = \cos\omega_s \cos\beta + \tan\delta \sin\beta \cos\gamma \tag{7}$$

$$C = \frac{\sin\beta \sin\gamma}{\cos\phi} \tag{8}$$

ϕ is the latitude (north positive)
δ is the solar declination
γ is the surface azimuth angle (i.e., the deviation of the normal to the surface from the local meridian with the zero point being due south, east negative and west positive)

Nomenclature

a = variable defined by equation (15)
a' = variable defined by equation (21)
A = variable defined by equation (6)
b = variable defined by equation (16)
B = variable defined by equation (7)
C = variable defined by equation (8)
d = variable defined by equation (17)
D = variable defined by equation (19)
G = function defined by equation (20)
$\bar{H}$ = monthly-average daily total radiation on a horizontal surface
$\bar{H}_d$ = monthly-average daily diffuse radiation on a horizontal surface
$\bar{H}_0$ = monthly-average daily extraterrestrial radiation
$\bar{H}_T$ = monthly-average daily total radiation on an inclined surface
I = instantaneous (or hourly) total radiation on a horizontal surface
I_d = instantaneous (or hourly) diffuse radiation on a horizontal surface
I_T = instantaneous (or hourly) total radiation on an inclined surface
$\bar{K}_T$ = ratio of monthly-average daily total to extraterrestrial radiation ($\bar{H}/\bar{H}_0$)
N = number of days sufficient to represent long-term average radiation distributions
r_d = long-term average ratio of instantaneous diffuse to daily diffuse radiation (equation (13))
r_T = long-term average ratio of instantaneous total to daily total radiation (equation (14))
$\bar{R}$ = ratio of monthly-average total radiation on an inclined surface to that on a horizontal surface ($\bar{H}_T/\bar{H}$)
$\bar{R}_b$ = ratio of monthly-average beam radiation on an inclined surface to that on a horizontal surface
t = time
t_{SR} = sunrise time for radiation on an inclined surface
t_{SS} = sunset time for radiation on an inclined surface
β = surface tilt from horizontal
γ = surface azimuth (0 deg = south, 90 deg = west, 180 deg = north)
δ = solar declination
θ_i = solar incidence angle of an inclined surface
θ_z = solar zenith angle
ρ = ground reflectance
ϕ = latitude
ω = solar hour angle; 15 deg × (hours from solar noon), afternoons positive
ω_s = sunset hour angle on a horizontal surface (equation (9))
ω_{SR} = sunrise hour angle for beam radiation on an inclined surface (equation (22))
ω_{SS} = sunset hour angle for beam radiation on an inclined surface (equation (23))

ω is the hour angle which is 15 deg × (hours from solar noon), afternoons positive, mornings negative)

ω_s is the sunset hour angle, defined by equation (9).

$$\omega_s = \arccos(-\tan\phi\tan\delta) \qquad (9)$$

The denominator of equation (3) is the product of $\bar{H}$, the monthly-average daily total radiation on a horizontal surface, and N, the number of days. As indicated by Collares-Pereira and Rabl [5], evaluation of the numerator of equation (3) is facilitated by exchanging the order of the integral and the summation. Substituting equation (4) for I_T, equation (3) can be rewritten as

$$\bar{R} = \frac{\int_{t_{SR}}^{t_{SS}} \left\{(\bar{I}-\bar{I}_d)R_b + \bar{I}_d\left(\frac{1+\cos\beta}{2}\right) + \rho\bar{I}\left(\frac{1-\cos\beta}{2}\right)\right\}dt}{\bar{H}} \qquad (10)$$

where the bars above I and I_d indicate long-term average values of these quantities obtained by summing I and I_d (for a specified time of day) over N days and then dividing by N.

Liu and Jordan [8] and Collares-Pereira and Rabl [9] have shown that $\bar{I}_d$ and $\bar{I}$ can be related to the long-term average daily diffuse radiation, $\bar{H}_d$, and the daily total radiation, $\bar{H}$, through the following relations.

$$\bar{I}_d = r_d\bar{H}_d \qquad (11)$$

$$\bar{I} = r_T\bar{H} \qquad (12)$$

where

$$r_d = \frac{\pi}{24d}[\cos\omega - \cos\omega_s] \qquad (13)$$

$$r_T = (a + b\cos\omega)r_d \qquad (14)$$

with

$$a = 0.409 + 0.5016\sin(\omega_s - 60\text{ deg}) \qquad (15)$$

$$b = 0.6609 - 0.4767\sin(\omega_s - 60\text{ deg}) \qquad (16)$$

$$d = \sin\omega_s - \frac{\pi}{180}\omega_s\cos\omega_s \qquad (17)$$

Substitution of equations (11) through (17) into equation (10) and integration over the period(s) between sunrise and sunset results in the following relation for $\bar{R}$.

$$\bar{R} = D + \frac{\bar{H}_D}{\bar{H}}\left(\frac{1+\cos\beta}{2}\right) + \rho\left(\frac{1-\cos\beta}{2}\right) \qquad (18)$$

where

$$D = \begin{cases} \text{MAX}\{0, G(\omega_{SS},\omega_{SR})\} & \text{if } \omega_{SS} \geq \omega_{SR} \\ \text{MAX}\{0, [G(\omega_{SS},-\omega_S) + G(\omega_S,\omega_{SR})]\} & \text{if } \omega_{SR} > \omega_{SS} \end{cases} \qquad (19)$$

$$G(\omega_1,\omega_2) = \frac{1}{2d}\left\{\left(\frac{bA}{2} - a'B\right)(\omega_1-\omega_2)\frac{\pi}{180} + (a'A - bB)(\sin\omega_1 - \sin\omega_2) - a'C(\cos\omega_1 - \cos\omega_2) + \frac{bA}{2}(\sin\omega_1\cos\omega_1 - \sin\omega_2\cos\omega_2) + \frac{bC}{2}(\sin^2\omega_1 - \sin^2\omega_2)\right\} \qquad (20)$$

$$a' = a - \frac{\bar{H}_d}{\bar{H}} \qquad (21)$$

The sunrise and sunset hour angles, ω_{SR} and ω_{SS}, are found by setting the numerator of equation (5) to zero (corresponding to a solar incidence angle of 90 deg) and solving for ω. A quadratic equation results, giving two values for ω which must be within $\pm\omega_S$ (the period between sunrise and sunset on a horizontal surface). The signs of ω_{SR} and ω_{SS} are affected by the surface orientation

$$|\omega_{SR}| = \text{MIN}\left\{\omega_S, \left[\arccos\left(AB + C\sqrt{A^2 - B^2 + C^2}\right)/(A^2 + C^2)\right]\right\}^* \qquad (22)$$

$$\omega_{SR} = \begin{cases} -|\omega_{SR}| & \text{if } (A>0 \text{ and } B>0) \text{ or } (A \geq B) \\ +|\omega_{SR}| & \text{otherwise} \end{cases}$$

$$|\omega_{SS}| = \text{MIN}\left\{\omega_S, \left[\arccos\left(AB - C\sqrt{A^2 - B^2 + C^2}\right)/(A^2 + C^2)\right]\right\}^* \qquad (23)$$

$$\omega_{SS} = \begin{cases} +|\omega_{SS}| & \text{if } (A>0 \text{ and } B>0) \text{ or } (A \geq B) \\ -|\omega_{SS}| & \text{otherwise} \end{cases}$$

Depending on the surface orientation and time of year, the sun may rise and set on an inclined surface once, twice, or not at all. For example, a north-facing vertical surface at 45 deg N latitude will receive no direct beam radiation (D = 0) during January. During July, beam radiation will rise on this north-facing surface at about 4:30 AM and set at 7:30 AM; it will rise a second time at 4:30 PM and finally set at 7:30 PM. In this case, D is found to be 0.074 by summing the radiation incident on the surface during the morning and afternoon periods, as indicated in equation (19). If this surface were facing south, rather than north, it would be irradiated by direct beam radiation from 7:30 AM to 4:30 PM in July and D would be 0.19. The algorithm given by equations (18) through (23) will result in an appropriate value of $\bar{R}$ in all of these situations. In addition, the algorithm is valid for negative latitudes (southern hemisphere) and for surface tilts greater than 90° or less than 0 deg.

A simpler algorithm can be derived for surfaces oriented directly toward the equator (in the northern hemisphere). In this case

$$\bar{R} = \frac{\cos(\phi-\beta)}{d\cos\phi}\left\{\left(a - \frac{\bar{H}_d}{\bar{H}}\right)\left(\sin\omega_s' - \frac{\pi}{180}\omega_s'\cos\omega_s''\right) + \frac{b}{2}\left(\omega_s'\frac{\pi}{180} + \sin\omega_s'(\cos\omega_s' - 2\cos\omega_s'')\right)\right\} + \frac{\bar{H}_d}{\bar{H}}\left(\frac{1+\cos\beta}{2}\right) + \rho\left(\frac{1-\cos\beta}{2}\right) \qquad (24)$$

where

$$\omega_S'' = \arccos(-\tan(\phi-\beta)\tan\delta) \qquad (25)$$

$$\omega_s' = \min(\omega_s, \omega_s'') \qquad (26)$$

(In the southern hemisphere, replace $(\phi-\beta)$ with $(\phi+\beta)$.)

*The quantity within the square root will be negative if the surface orientation is such that the solar incidence angle is less or greater than 90 deg at all times; ω_{SR} and ω_{SS} should be set to $-\omega_S$ and $+\omega_S$, respectively, under these circumstances.

3 Results and Discussion

Two methods for estimating $\bar{R}$ (equation (2) and equation (18)) have been described. Both algorithms rely on a number of assumptions concerning the long-term average distributions of radiation. Both algorithms assume that radiation is symmetrically distributed about solar noon, and that diffuse and ground-reflected radiation are isotropic. The only difference between the two algorithms is in the manner in which the beam radiation component is treated. The Liu and Jordan method assumes that $\bar{R}_b$ can be estimated as the ratio of monthly-average extraterrestrial radiation on the inclined surface to that on a horizontal surface. This manner of calculating $\bar{R}_b$ neglects the fact that the long-term average ratio of instantaneous beam to extraterrestrial radiation varies with the time of day, as indicated by equations (13) and (14). The algorithm given by equation (18) uses this information.

Both algorithms require an estimate of $\bar{H}_d/\bar{H}$, the monthly-average diffuse radiation fraction. Experimental results have shown that $\bar{H}_d/\bar{H}$ is related to $\bar{K}_T$, the ratio of average daily total to extraterrestrial radiation. Correlations of $\bar{H}_d/\bar{H}$ to $\bar{K}_T$ have been developed by Liu and Jordan [8], Page [4], Tuller [10], Collares-Pereira and Rabl [9], and others. These correlations are not in good agreement; it is not possible at this time to determine which of these correlations will result in the most reliable estimates of $\bar{R}$. The Liu and Jordan correlation is used in the results which follow.

Long-term measurements of radiation data on both inclined and horizontal surfaces are not available in sufficient quantity at this time to permit an evaluation of the $\bar{R}$ algorithms described in sections 1 and 2. Even if such data were available, the advantage of estimating the beam radiation component as described in section 2 would be masked by uncertainties in the diffuse and ground-reflected components.

The advantage of the algorithm in section 2 is best shown by comparison of the results of both algorithms with values of $\bar{R}$ determined from numerical integration of equation (3) (which defines $\bar{R}$). Twenty-three years of hourly radiation measurements in Madison, Wis., Albuquerque, N. Mex., and Miami, Fla. were used for this purpose [11]. Quarter hour time steps were used in the integration to minimize the effect, observed by Hay, of a "poorly defined" day length. Radiation on the inclined surface was estimated using equation (4) with the ground reflectance set to 0.2. It should be noted that only total radiation data on the horizontal surface were available; the hourly diffuse radiation fraction, I_d/I, was estimated using the Liu and Jordan [8] correlation. The hourly diffuse fraction correlation is analogous to (but different from) the monthly-average diffuse radiation fraction. The hourly and monthly-average diffuse radiation correlations should be related as indicated in equation (27). The two Liu and Jordan correlations are consistent in this manner provided that the long-term distribution (i.e., the relative numbers of poor, average and excellent days) of radiation is consistent with the Liu and Jordan distribution given in Fig. 9 of reference [8].

$$\frac{\bar{H}_d}{\bar{H}} = \frac{\sum_{\mathrm{day}=1}^{N} \int_{t_{SR}}^{t_{SS}} \left(\frac{I_d}{I}\right) I\,dt}{\sum_{\mathrm{day}=1}^{N} \int_{t_{SR}}^{t_{SS}} I\,dt} \tag{27}$$

The alternative methods of calculating $\bar{R}$ are compared in Tables 1, 2, and 3 (Madison, Albuquerque, and Miami) for vertical surfaces facing south, west, and east. (The subscripts associated with $\bar{R}$ in the column headings identify the equation used to calculate $\bar{R}$.) For the south-facing surfaces, the values of $\bar{R}$ computed using equation (18) are always closer to unity than those computed from equation (2) in all three locations; i.e., $\bar{R}_{18}$ is lower than $\bar{R}_2$ during the winter and slightly higher during the summer. During the equinox months (March and September), the values of $\bar{R}_2$ and $\bar{R}_{18}$ are nearly identical as expected since an examination of equation (8) of reference [2] and equation (24) shows that the beam radiation component for both algorithms reduces to $[\cos(\phi-\beta)/\cos\phi][1-\bar{H}_d/\bar{H}]$ for surfaces facing directly toward the equator during the equinox ($\delta=0$). In general, the agreement between values of $\bar{R}_{18}$ and $\bar{R}_3$ is better than that between $\bar{R}_2$ and $\bar{R}_3$.

The algorithms given by equation (2) and equation (18) both assume that radiation is symmetrically distributed about solar noon. As a result, these algorithms result in the same values of $\bar{R}$ for surfaces facing east as they do for surfaces facing west. The effect of asymmetry can be seen in Tables 1-3 by comparing values of $\bar{R}_3^+$ and $\bar{R}_3^-$, the values of $\bar{R}$ determined by numerical integration of equation (3) for the west and east facing surfaces, respectively. In all three locations, values of $\bar{R}_3^+$ and $\bar{R}_3^-$ are nearly identical for November through March. In Albuquerque and Miami, however, there

Table 1 $\bar{R}$ values for vertical surfaces in Madison, Wis. (Lat. 43 deg N)

	$\bar{K}_T$	$\gamma = 0°$			$\gamma = \pm 90°$			
		$\bar{R}_2$	$\bar{R}_3$	$\bar{R}_{18}$	$\bar{R}_2$	$\bar{R}_3^+$	$\bar{R}_3^-$	$\bar{R}_{18}$
JAN	0.44	1.83	1.72	1.73	0.81	0.70	0.69	0.71
FEB	0.49	1.40	1.39	1.33	0.78	0.69	0.70	0.68
MAR	0.50	0.95	0.96	0.94	0.73	0.63	0.66	0.64
APR	0.47	0.63	0.65	0.64	0.68	0.57	0.51	0.60
MAY	0.51	0.47	0.48	0.49	0.65	0.55	0.57	0.59
JUN	0.53	0.41	0.42	0.42	0.64	0.53	0.55	0.58
JUL	0.55	0.43	0.45	0.45	0.65	0.54	0.55	0.59
AUG	0.55	0.55	0.57	0.57	0.67	0.55	0.57	0.60
SEP	0.52	0.81	0.82	0.81	0.71	0.58	0.61	0.62
OCT	0.50	1.24	1.21	1.19	0.76	0.63	0.66	0.66
NOV	0.39	1.59	1.57	1.50	0.78	0.67	0.70	0.68
DEC	0.38	1.85	1.77	1.73	0.80	0.68	0.71	0.71

Table 2 $\bar{R}$ values for vertical surfaces in Albuquerque N.Mex. (Lat. 35 deg N)

	$\bar{K}_T$	$\gamma = 0°$			$\gamma = \pm 90°$			
		$\bar{R}_2$	$\bar{R}_3$	$\bar{R}_{18}$	$\bar{R}_2$	$\bar{R}_3^+$	$\bar{R}_3^-$	$\bar{R}_{18}$
JAN	0.64	1.63	1.55	1.55	0.77	0.66	0.66	0.68
FEB	0.67	1.23	1.20	1.18	0.73	0.64	0.64	0.65
MAR	0.68	0.83	0.82	0.82	0.69	0.60	0.62	0.61
APR	0.71	0.51	0.53	0.52	0.66	0.55	0.58	0.59
MAY	0.73	0.34	0.36	0.35	0.63	0.51	0.56	0.57
JUN	0.74	0.28	0.30	0.29	0.62	0.50	0.55	0.56
JUL	0.70	0.32	0.33	0.32	0.62	0.47	0.55	0.56
AUG	0.70	0.43	0.46	0.44	0.65	0.50	0.56	0.58
SEP	0.71	0.69	0.70	0.69	0.68	0.55	0.60	0.60
OCT	0.71	1.11	1.07	1.07	0.72	0.61	0.64	0.64
NOV	0.67	1.54	1.48	1.47	0.76	0.66	0.67	0.67
DEC	0.63	1.76	1.68	1.67	0.78	0.66	0.67	0.69

Table 3 $\bar{R}$ values for vertical surfaces in Miami, Fla. (Lat. 25.8 deg N)

	$\bar{K}_T$	$\gamma = 0°$			$\gamma = \pm 90°$			
		$\bar{R}_2$	$\bar{R}_3$	$\bar{R}_{18}$	$\bar{R}_2$	$\bar{R}_3^+$	$\bar{R}_3^-$	$\bar{R}_{18}$
JAN	0.51	1.16	1.08	1.10	0.68	0.59	0.59	0.60
FEB	0.54	0.92	0.88	0.88	0.66	0.57	0.58	0.59
MAR	0.56	0.65	0.64	0.64	0.64	0.55	0.56	0.57
APR	0.57	0.41	0.43	0.42	0.62	0.52	0.53	0.55
MAY	0.53	0.31	0.33	0.32	0.61	0.51	0.54	0.54
JUN	0.48	0.30	0.31	0.30	0.60	0.48	0.54	0.54
JUL	0.50	0.30	0.32	0.31	0.60	0.48	0.54	0.54
AUG	0.49	0.38	0.41	0.38	0.61	0.48	0.53	0.55
SEP	0.49	0.55	0.56	0.55	0.63	0.51	0.56	0.56
OCT	0.52	0.82	0.78	0.79	0.65	0.55	0.57	0.58
NOV	0.52	1.10	1.02	1.04	0.67	0.58	0.58	0.60
DEC	0.52	1.25	1.15	1.18	0.69	0.59	0.59	0.61

are significant differences between the values of $\bar{R}$ for the east and west facing surfaces during April through October. In both locations, values of $\bar{R}_3^-$ are higher than $\bar{R}_3^+$, indicating that there is more solar radiation in the morning than in the afternoon.

Tables 1-3 also show that, for the east and west facing surfaces, the agreement between $\bar{R}_{18}$ and $\bar{R}_3^-$ (or $\bar{R}_3^+$) is significantly better than that between $\bar{R}_2$ and $\bar{R}_3^-$ (or $\bar{R}_3^+$) for all months in all three locations. These results indicate that the estimation of $\bar{R}_b$ as the ratio of monthly average extraterrestrial radiation on an inclined surface to that on a horizontal surface can be significantly in error for surfaces facing east or west of south.

4 Conclusion

The widespread utilization of solar energy will depend on computationally simple, yet accurate design methods, such as the *f*-chart [12], Solar Load Ratio [13], and utilizability [6,14] methods. These design methods require, as input data, estimates of $\bar{H}_T$, the monthly-average daily radiation on inclined surfaces. A method for calculating $\bar{H}_T$ on a surface of any orientation is presented in this paper. This method differs from the Liu and Jordan [1,2] method in the manner in which the monthly-average beam radiation is determined; the method presented here yields values of $\bar{H}_T$ which agree more closely with integrated hourly calculations, especially for surfaces facing east or west of south.

5 Example

Estimate the monthly-average daily radiation incident on vertical surfaces oriented south, north and west for July in Madison, Wis. The monthly-average daily radiation on a horizontal surface, $\bar{H}$, is 22.1 MJ/m^2-day. The ground reflectance is 0.2.

The monthly-average daily extraterrestrial radiation, $\bar{H}_0$ is found to be 40.1 MJ/m^2-day from either equation (3) or Table 1 of reference [2]. $\bar{K}_T$ is the ratio $\bar{H}/\bar{H}_0$ or 0.55. The monthly-average diffuse fraction is then 0.33 using the Liu and Jordan relationship given in equation (10*a*) of reference [2].

The average solar declination, δ, is 21.2 deg in July. The latitude, ϕ, is 43 deg for Madison. From equation (9), the sunset hour angle, ω_s is then 111.2 deg. From equations (15), (16), (17), and (21), $a = 0.800$, $b = 0.289$, $d = 1.634$, and $a' = 0.472$.

The remainder of the calculations are affected by the orientation of the surface. The calculation order, equation numbers, and values of intermediate variables are summarized in Table 4 for the south, north, and west facing surfaces. Also given in Table 4 are the resulting values of $\bar{R}$ and $\bar{H}_T$, the monthly-average daily radiation on the inclined surfaces.

Table 4 Intermediate results in the calculation of $\bar{R}$

		$\gamma = 0°$	$\gamma = 180°$	$\gamma = 90°$
A	[Equation (6)]	0.933	-0.933	0.0
B	[Equation (7)]	0.388	-0.388	0.0
C	[Equation (8)]	0.0	0.0	1.37
ω_{SR}	[Equation (22)]	-65.4	65.4	0
ω_{SS}	[Equation (23)]	65.4	-65.4	111.2
D	[Equation (19)]	0.180	0.071	0.32
$\bar{R}$	[Equation (18)]	0.44	0.34	0.59
$\bar{H}_T$	[MJ/m^2-day]	9.72	7.57	13.0

Acknowledgment

The research of which this study is a part has been financially supported by the Department of Energy, Contract #EY-76-S-02-2588.A003.

References

1 Liu, B.Y.H. and Jordan, J.C., "Daily Insolation on Surfaces Tilted Towards the Equator," *Trans. ASHRAE*, 1962, pp. 526–541.

2 Klein, S.A., "Calculation of Monthly Average Insolation on Tilted Surfaces," *Solar Energy*, Vol. 19, 1977, pp. 325–329.

3 Hay, J.E., "Calculation of the Monthly Mean Solar Radiation for Horizontal and Inclined Surfaces," *Solar Energy*, Vol. 23, 1979, pp. 301–307.

4 Page, J.K., "The Estimation of Monthly Mean Values of Daily Total Short-Wave Radiation on Vertical and Inclined Surfaces from Sunshine Records for Latitudes 40° N-40°S," *Proceedings of the UN Conference on New Sources of Energy*, paper no. 35/5/98, 1961.

5 Collares-Pereira, M., and Rabl, A., "Derivation of Method for Predicitng Long-Term Average Energy Delivery of Solar Collectors," *Solar Energy*, Vol. 23, 1979, pp. 223–233.

6 Collares-Pereira, M., and Rabl, A., "Procedure for Predicting Performance of Solar Collectors," *Solar Energy*, Vol. 23, 1979, pp. 235–253.

7 Duffie, J.A., and Beckman, W.A., *Solar Engineering of Thermal Processes*. Wiley Interscience, 1980.

8 Liu, B.Y.H., and Jordan, R.C., "Availability of Solar Energy for Flat-Plate Solar Heat Collection," Chapter 5, Applications of Solar Energy for Heating and Cooling of Buildings, ASHRAE GRP 170, 1977.

9 Collares-Pereira, M., and Rabl, A., "The Average Distribution of Solar Radiation - Correlations Between Diffuse and Hemisperhical and Between Daily and Hourly Insolation Values," *Solar Energy*, Vol. 22, No. 2, 1979.

10 Tuller, S.E., "The Relationship Between Diffuse, Total, and Extra-terrestrial Radiation," *Solar Energy*, Vol. 18, No. 3, 1976.

11 SOLMET, Hourly Solar Radiation-Surface Meteorological Observations, NOAA, volume 2, TD-9724, 1979.

12 Beckman, W.A., Klein, S.A., and Duffie, J.A., *Solar Heating Design*, Wiley-Interscience, 1977.

13 Balcomb, J.D., and McFarland, R.D., "A Simple Empirical Method for Estimating the Performance of a Passive Solar Heated Building of the Thermal Storage Wall Type," *Proceedings of the 2nd National Passive Solar Conference*, Philadelphia, Pa., Mar. 16-18, 1978.

14 Klein, S.A., "Calculation of Flat-Plate Collector Utilizability," *Solar Energy*, Vol. 21, 1978, pp. 393–402.