

the transition region than in the  $n$ -region; the effective length of flow in the  $n$ -region, being  $L_p$ , is greater than the width of the transition region. Consequently, the variation of  $\varphi_p$  shown in Fig. 6(c) is seen to be reasonable. Similar considerations apply to  $\varphi_n$ . As is shown in Fig. 6(c), the application of  $\delta\varphi$  does not alter  $\varphi_p - \psi$  in the  $p$ -region nor  $\varphi_n - \psi$  in the  $n$ -region. The

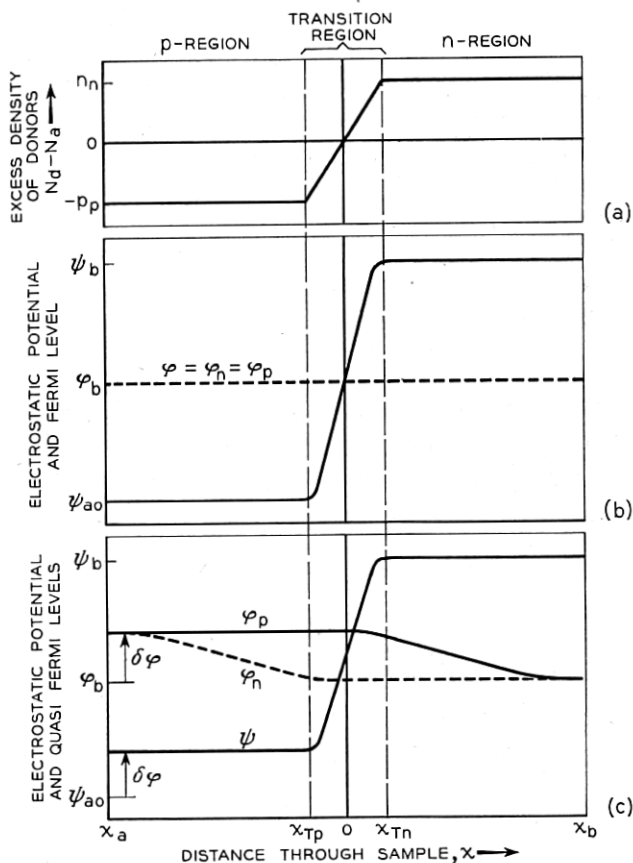


Fig. 6—Simplified model of a  $p$ - $n$  junction.

- (a) Distribution of donors and acceptors.
- (b) Potentials for thermal equilibrium.
- (c) Effect of  $\delta\varphi$  applied potential in forward direction.

reason, as discussed in connection with (2.31), is that in these regions electrical neutrality requires an essentially constant value for the more abundant carrier. Hence the relationships between the  $\varphi$ 's and  $\psi$  follow from (2.4).

The nature of the potential distribution in the transition region has no effect in the considerations just discussed. However, as shown in Section 2,

the capacity of the transition region, which we shall denote by  $C_T$  in this section, does depend on the nature of the transition region and, consequently, on the value of  $K$ .

If the sizes of the  $p$ -region and  $n$ -region are large compared to the diffusion lengths, we may assume the current at  $x_a$  to be substantially  $I_p$  only and that at  $x_b$ ,  $I_n$  only. The total current entering at  $x_a$  can be accounted for as doing three things: (1) neutralizing the electron current flowing into the  $p$ -region across  $x_{Tp}$ , (2) contributing to the charge in the transition region (this corresponds to the capacity discussed in Section 2) and (3) contributing a current flow to the right across  $x_{Tn}$ .

We have selected the hole current for analysis because the hole has a positive charge and the connection between the algebra and the physical picture is simplified. For the same reason, the text emphasizes forward current, although the equations are equally applicable to reverse currents. Nothing essential is left out by this process; since the sample as a whole remains uncharged, the current  $I$  is the same for all values of  $x$  and if  $I_p$  is known, then  $I_n = I - I_p$  is also determined.

#### 4.2 Solution for Hole Flow into the $n$ -region

We shall calculate first the hole current  $I_p(x_{Tn})$  flowing across  $x_{Tn}$ . It is readily evaluated as follows: The value of  $p(x_{Tn})$  is given by

$$\begin{aligned} p(x_{Tn}) &= n_i e^{q(\varphi_b + \delta\varphi - \psi_b)/kT} \\ &= p_n e^{q\delta\varphi/kT} \end{aligned} \quad (4.1)$$

where  $p_n$  is the hole concentration in the  $n$ -region for thermal equilibrium. If we apply a small a-c. signal superimposed on a d-c. bias so that

$$\delta\varphi = v_0 + v_1 e^{i\omega t} \quad (4.2)$$

where  $v_1$  is an a-c. signal, assumed so small that linear theory may be employed (i.e.  $v_1 \ll kT/q$ ), then

$$p(x_{Tn}) = (p_n e^{qv_0/kT})(1 + (qv_1/kT)e^{i\omega t}).$$

We resolve this density into a d-c. component  $p_0$  and an a-c. component  $p_1 e^{i\omega t}$ :

$$p(x_{Tn}) = p_n + p_0 + p_1 e^{i\omega t} \quad (4.3)$$

where

$$p_0 = p_n(e^{qv_0/kT} - 1) \quad (4.4)$$

$$p_1 = (qp_n v_1/kT)e^{qv_0/kT}. \quad (4.5)$$

So long as  $p(x_{Tn}) \ll n_n$ , the normal concentration of electrons in the

$n$ -region, the lifetime  $\tau_p$  and diffusion constant  $D$  for a hole will be substantially unaltered by  $\delta\varphi$ . Application of the hole-current equation to the hole density  $p(x, t)$  gives

$$I_p = -qD \frac{\partial p}{\partial x}. \quad (4.6)$$

Combining this with the recombination equation

$$\frac{\partial p}{\partial t} = \frac{p_n - p}{\tau_p} - \frac{1}{q} \frac{\partial I_p}{\partial x} = \frac{p_n - p}{\tau_p} + D \frac{\partial^2 p}{\partial x^2} \quad (4.7)$$

leads to the solution

$$p = p_n + p_0 e^{(x_{Tn}-x)/\sqrt{D\tau_p}} + p_1 e^{i\omega t + (x_{Tn}-x)(1+i\omega\tau_p)^{1/2}/(D\tau_p)^{1/2}}. \quad (4.8)$$

The quantity  $\sqrt{D\tau_p}$  is the diffusion length and is denoted by  $L_p$ . (We shall use subscript  $p$  for holes in the  $n$ -region and  $n$  for electrons in the  $p$ -region for both  $L$  and  $\tau$ .)

When  $p$  is large compared to  $p_n$ , but small compared to  $n_n$ , the expression for  $p$  leads to the following formula for  $\varphi_p$ :

$$\varphi_p = \varphi_n + v_c - (kT/q)(x - x_{Tn})/L_p + v_1 e^{i\omega t - (x - x_{Tn})[(1+i\omega\tau_p)^{1/2}-1]/L_p}. \quad (4.9)$$

This shows that the d-c. part of  $\varphi_p$  varies linearly in the  $n$ -region, for large forward currents, and decreases by  $(kT/q)$  in each diffusion length  $L_p$ . The transition from this linear dependence to an exponential decay for  $\varphi_p$  comes when  $\varphi_p - \varphi_n = (kT/q)$ . This behavior of the d-c. part of  $\varphi_p$  is useful in connection with diagrams of  $\varphi_p$  versus distance. (See Sections 5 and 6.)

The solution just obtained for  $p$  gives rise to a current at  $x_{Tn}$  of

$$\begin{aligned} I_p(x_{Tn}) &= -qD \frac{\partial p}{\partial x} \\ &= qp_0 D/L_p + qp_1 D e^{i\omega\tau} (1 + i\omega\tau_p)^{1/2}/L_p. \end{aligned} \quad (4.10)$$

The d-c. part is calculated by substituting (4.4) for  $p_0$ :

$$\begin{aligned} I_{p0}(x_{Tn}) &= (qp_n D/L_p)(e^{qv_0/kT} - 1); \\ &\equiv I_{ps}(e^{qv_0/kT} - 1) \end{aligned} \quad (4.11)$$

and the a-c. part is similarly obtained from (4.5) for  $p_1$ :

$$\begin{aligned} I_{p1}(x_{Tn}) &= (qp_n \mu/L_p)[e^{(qv_0/kT)}](1 + i\omega\tau_p)^{1/2} v_1 e^{i\omega t} \\ &\equiv (G_p + iS_p)v_1 e^{i\omega t} \equiv A_p v_1 e^{i\omega t} \end{aligned} \quad (4.12)$$

where  $A_p$  is called the admittance (per unit area) for holes diffusing into the  $n$ -region; its real and imaginary parts are the conductance and suscept-

ance. For  $\omega\tau_p$  small, the real term  $G_p$  is simply conductance per  $\text{cm}^2$  of a layer  $L_p$  cm thick with hole conduction corresponding to the density  $p_n + f_0$ ; it is also the differential conductance obtained by differentiating (4.11) in respect to  $v_0$ . For the case of zero bias this establishes the result quoted in Section 1 that the voltage drop is due to hole flow in the  $n$ -region where the hole conductivity is low.

In this section we have treated  $\tau_p$  as arising from body recombination. In a sample whose  $y$  and  $z$  dimensions are comparable to  $L_p$  or  $L_n$ , surface recombination may play a dominant role. However, as we show in Appendix V, the theory given here may still apply provided appropriate values for  $\tau_p$  and  $\tau_n$  are used.

### 4.3 D-C. Formulae

The total direct hole current flowing in at  $x_a$  is  $I_{x0}$  plus the current required to recombine with electrons in the  $p$ -region. This latter current is, of course, equal to the electron current flowing into the  $p$ -region. This electron current, denoted by  $I_{x,0}$  or  $I_{x,0}(x_{Tp})$ , is obtained by the same procedure as that leading to (4.11) for  $I_{x,0}$  except that  $bD$  replaces  $D$  and the subscripts of  $L$  and  $\tau$  are now  $n$ . Combining the two currents leads to the total direct current:

$$I_0 = I_{p0} + I_{n0} = (qL) \left( \frac{p_n}{L_p} + \frac{bn_p}{L_n} \right) (e^{qv_0/kT} - 1) \quad (4.13)$$

for the direct current per unit area for applied potential  $v_0$ .<sup>12</sup> The algebraic signs are such that  $I > 0$  corresponds to current from the  $p$ -region to the  $n$ -region in the specimen;  $v_0 > 0$  corresponds to a plus potential applied to the  $p$ -end. The ratio of hole current to electron current across the transition region is

$$\begin{aligned} \frac{I_{p0}}{I_{n0}} &= \frac{p_n}{L_p} \cdot \frac{L_n}{bn_p} = \frac{p_p}{bn_n} \cdot \frac{\sqrt{bD\tau_n}}{\sqrt{D\tau_p}} \\ &= \frac{p_p}{bn_n} \sqrt{\frac{bn_n}{p_p}} = \sqrt{\frac{\sigma_p}{\sigma_n}} \end{aligned} \quad (4.14)$$

where we have used the relationships  $n_n p_n = n_p p_p = n_i^2$  from (2.2) and  $\tau_p n_n = \tau_n p_p = 1/r$  from (3.2) and (3.3). These results can be summarized by saying that the current flows principally into the material of higher re-

<sup>12</sup> For convenience we repeat the definitions here:  $q \equiv$  magnitude of electronic charge;  $D \equiv$  diffusion constant for holes;  $p_n$  and  $n_n \equiv$  thermal equilibrium value of  $p$  and  $n$ , assumed constant throughout  $n$ -region ( $x > x_{Tp}$ );  $n_p$  and  $p_p \equiv$  similar values for  $x < x_{Tp}$ ;  $L_p \equiv$  diffusion length  $\equiv \sqrt{D\tau_p}$  for holes in  $n$ -region;  $\tau_p \equiv$  lifetime of hole in  $n$ -region before recombination;  $b =$  electron mobility/hole mobility;  $L_n$  and  $\tau_n$  similar in quantities for electrons in  $p$ -region;  $\sigma_n = q\mu b n_n$  and  $\sigma_p = q\mu p_p$  are the conductivities of the two regions.

sistivity. We can also say that the hole current depends only on the *n*-type material and vice versa. For a *p-n* junction emitter in a transistor with an *n*-type base, it is thus advantageous to use high conductivity *p*-type material so as to suppress an unwanted electron current.

For comparison with experiment, it is advantageous to express the values of  $p_n$  and  $n_p$  in terms of the conductivities  $\sigma_n$  and  $\sigma_p$ . If the conductivity of the intrinsic material is written as

$$\sigma_i = q\mu n_i(1 + b), \quad (4.15)$$

then, if  $p_n \ll n_n$  and  $n_p \ll p_p$ , we find

$$q\mu p_n = b\sigma_i^2/(1 + b)^2\sigma_n \quad (4.16)$$

$$q\mu b n_p = b\sigma_i^2/(1 + b)^2\sigma_p. \quad (4.17)$$

Using these equations, we may rewrite (4.11) and a corresponding equation for electron current into the *p*-region so as to express their dependence on d-c. bias  $v_0$  and the properties of the regions:

$$\begin{aligned} I_{p0}(v_0) &= \frac{b\sigma_i^2}{(1 + b)^2\sigma_n L_p} \cdot \frac{kT}{q} (e^{qv_0/kT} - 1) \\ &\equiv G_{p0} \frac{kT}{q} (e^{qv_0/kT} - 1) \\ &\equiv I_{ps}(e^{qv_0/kT} - 1) \end{aligned} \quad (4.18)$$

$$\begin{aligned} I_{n0}(v_0) &= \frac{b\sigma_i^2}{(1 + b)^2\sigma_p L_n} \cdot \frac{kT}{q} (e^{qv_0/kT} - 1) \\ &\equiv G_{n0} \frac{kT}{q} (e^{qv_0/kT} - 1) \\ &\equiv I_{ns}(e^{qv_0/kT} - 1). \end{aligned} \quad (4.19)$$

The values of  $G_{p0}$  and  $G_{n0}$  (which are readily seen to be the values of the low-frequency, low-voltage ( $v_0 < kT/q$ ) conductances) and the saturation reverse currents are given by

$$G_{p0} \equiv \frac{b\sigma_i^2}{(1 + b)^2\sigma_n L_p} \equiv \frac{q}{kT} I_{ps} \quad (4.20)$$

$$G_{n0} \equiv \frac{b\sigma_i^2}{(1 + b)^2\sigma_p L_n} \equiv \frac{q}{kT} I_{ns} \quad (4.21)$$

The expression for direct current then becomes

$$\begin{aligned} I_0(v_0) &= [I_{p0} + G_{n0}] \left( \frac{kT}{q} \right) [e^{qv_0/kT} - 1] \\ &= (I_{ps} + I_{ns}) [e^{qv_0/kT} - 1]. \end{aligned} \quad (4.22)$$

#### 4.4 Total Admittance

In order to calculate the alternating current, we must include the capacity of the transition region, discussed in Section 2. Denoting this by  $C_T$ , we then find for the total alternating current.

$$I_{ac} = (G_p + iS_p + G_n + iS_n + i\omega C_T) v_1 = A v_1 \quad (4.23)$$

where  $G_n$  and  $S_n$  are similar to  $G_p$  and  $S_p$  but apply to electron current into the  $p$ -region. The value of the hole and electron admittances can be expressed as

$$A_p = G_p + iS_p = (1 + i\omega\tau_p)^{1/2} G_{p0} e^{qv_0/kT} \quad (4.24)$$

$$A_n = G_n + iS_n = (1 + i\omega\tau_n)^{1/2} G_{n0} e^{qv_0/kT} \quad (4.25)$$

For low frequencies, such that  $\omega$  is much less than  $1/\tau_p$ , we can expand  $G_p + iS_p$  as follows:

$$G_p + iS_p = G_{p0} e^{qv_0/kT} + i\omega(\tau_p/2) G_{p0} e^{qv_0/kT} \quad (4.26)$$

Hence  $(\tau_p/2) G_{p0} e^{qv_0/kT}$  behaves like a capacity.

It is instructive to interpret this capacity for the case of zero bias,  $v_0 = 0$ , for which we find:

$$C_p = \tau_p G_{p0}/2 = \tau_p q p_n \mu / 2 L_p = q^2 p_n L_p / 2 k T. \quad (4.27)$$

The last formula, obtained by noting that  $\tau_p \mu = q \tau_p D / k T = q L_p^2 / k T$ , has a simple interpretation:  $q p_n L_p$  is the total charge of holes in a layer  $L_p$  thick. For a small change in voltage  $v$ , this density should change by a fraction  $qv/kT$  so that the change in charge divided by the change in  $v$  is  $(q/kT)(q p_n L_p)$  which differs from  $C_p$  only by a factor of 2, which arises from the nature of the diffusion equation.

This capacity can be compared with  $C_{T \text{ neut.}}$ , discussed in Section 2, (see equation (2.39) and text for (2.42)) for germanium at room temperature as follows:

$$\frac{C_p}{C_{T \text{ neut.}}} = \frac{q^2 p_n L_p}{2 k T} \cdot \frac{k T a}{10 q^2 n_i^2} = \frac{p_n L_p a}{20 n_i^2}. \quad (4.28)$$

For a structure like Fig. 6(c), the excess of donors over acceptors reaches its maximum value, equal to  $n_n$ , at  $x_{Tn}$  leading to  $n_n = a x_{Tn}$ . Consequently  $a = n_n / x_{Tn}$ . Substituting this value for  $a$  in (4.28) and noting that  $p_n n_n = n_i^2$  gives

$$\frac{C_p}{C_{T \text{ neut.}}} = \frac{L_p}{20 x_{Tn}} \quad (4.29)$$

As discussed at the beginning of this section,  $L_p \doteq 6 \times 10^{-3}$  cm for holes

in germanium. Hence if the transition region is  $6 \times 10^{-4}$  cm thick, the diffusion capacity  $C_p$  will dominate the capacitive term in the admittance.

Although  $A_p$  simulates a conductance and capacitance in parallel at low frequencies, its high-frequency behavior is quite different. In Fig. 7 the

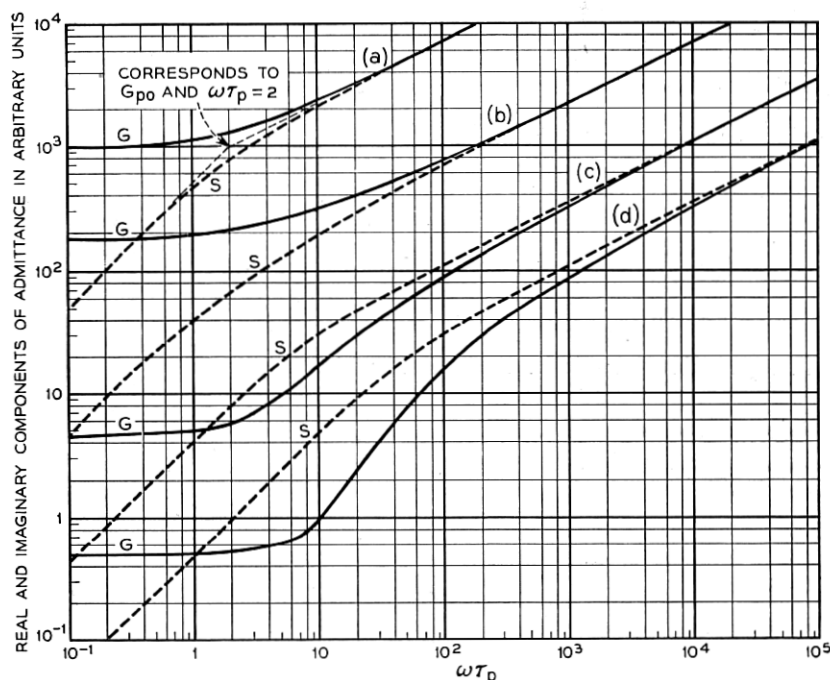


Fig. 7—Real,  $G$ , and imaginary,  $S$ , components of admittance for hole flow into  $n$ -region.

(a)  $10^3 A_p / G_{p0} = 10^3 (1 + i\omega\tau_p)^{1/2}$  corresponding to uniform  $n$ -region.

(b)  $10^2 \times$  Formula of Appendix III, corresponding to layer of high recombination rate in front of  $n$ -region. This causes  $G$  to exceed  $S$  at higher frequencies than for (a).

(c)  $10 \times$  Equation (4.33), corresponding to a retarding field in the  $n$ -region, with  $L_r = L_p / \sqrt{10}$ .

(d) Equation (4.33) with  $L_r = L_p / 10$ .

behavior of  $(1 + i\omega\tau_p)^{1/2} = A_p / G_{p0}$ , is shown. For high frequencies  $G_p$  and  $S_p$  are equal:

$$G_p = S_p = \sqrt{\tau_p / 2} G_{p0} \sqrt{\omega} = \frac{b\sigma_i^2 \sqrt{\omega}}{(1 + b)^2 \sigma_n \sqrt{2D}} \quad (4.30)$$

Thus for high frequencies the admittance is independent of  $\tau_p$  and is determined by the diffusion of holes in and out of the  $n$ -region. The three straight asymptotes have a common intersection at the point  $G_{p0}$ ,  $\omega\tau = 2$  on Fig. 7, a fact which is useful in estimating the value of  $\tau$  from such data.

For large  $\omega$ ,  $S_p$  varies as  $\omega^{1/2}$  as shown in (4.30) whereas  $S_T = \omega C_T$ . Hence

at very high frequencies  $C_T$  will dominate the admittance. At very high frequencies  $C_T$  itself will have a frequency dependence; however, for the assumptions on which the treatment of this section is based, the relaxation time for the transition region  $\tau_T$  is much less than  $\tau_p$ . This is a consequence of the fact that, although diffusion of holes into the transition region is required for the charging of  $C_T$ , the distance is relatively short, being in fact only that fraction of the width  $x_{Tn} - x_{Tp}$  of the transition region in which  $\psi$  rises by  $kT/q$ ; in germanium this will be about one-tenth of  $x_{Tn} - x_{Tp}$ . Since diffusion times vary as (distance)<sup>2</sup>, the ratio of the times is

$$\frac{\tau_T}{\tau_p} = \frac{(x_{Tn} - x_{Tp})^2}{100L_p^2}. \quad (4.31)$$

Hence if  $L_p > x_{Tn} - x_{Tp}$ ,  $\tau_T$  will be much less than  $\tau_p$ .<sup>13</sup>

#### 4.5 Admittance Due to Hole Flow in a Retarding Field

In Appendix II we treat the case in which a potential gradient, due to changing concentration for example, is present in the  $n$ - and  $p$ -regions. This tends to prevent holes from diffusing deep in the  $n$ -region and for this reason the  $n$ -region acts partly like a storage tank for holes under a-c. conditions, thus enhancing  $S_p$  compared to  $G_p$  in  $A_p$ . If the electric field is  $-d\psi/dx = kT/qL_r$ , where  $L_r$  is the distance required for an increase of  $kT/q$  of potential (i.e. a factor of  $e$  increase in  $n_n$ ), then the value of  $A_p$  is

$$A_p = [q\mu p_n/L_p] \frac{(2L_r/L_p)(1 + i\omega\tau_p)}{1 + [1 + (1 + i\omega\tau_p)(2L_r/L_p)^2]^{1/2}} \quad (4.32)$$

For  $\omega\tau_p > 1$ , this admittance is largely reactive provided  $2L_r/L_p$  is sufficiently small.

The dependence of  $A_p$  upon  $\omega$  is shown in Fig. 7 for two values of  $L_r/L_p$ . The plot shows the real and imaginary parts of

$$A_p/[2q\mu p_n L_r/L_p^2] = \frac{(1 + i\omega\tau_p)}{1 + [1 + (1 + i\omega\tau_p)(2L_r/L_p)^2]^{1/2}} \quad (4.33)$$

for  $L_p/L_r = 10^{1/2}$  and  $L_p/L_r = 10$ , the two curves being relatively displaced vertically by one decade. The second value implies that the field keeps the holes back so that they penetrate only  $\frac{1}{10}$  their possible diffusion length in no field. It is seen that for this case the storage effect is very pronounced and the susceptance  $S$  is much larger than  $G$  for high frequencies.

The function  $(1 + i\omega\tau_p)^{1/2}$ , discussed earlier, corresponds to the limiting case of (4.32) for  $L_r = \infty$ .

<sup>13</sup> In Appendix IV an analytic treatment of  $C_T$  is given.



#### 4.6 The Effect of a Region of High Rate of Generation

There is evidence that imperfections, such as surfaces and cracks, add materially to the rate of generation and recombination of holes and electrons. If there is a localized region of high recombination rate in the transition region, there will be a pronounced modification of the admittance characteristics. In Fig. 8(a) such a layer is represented at  $x = 0$ . In Fig. 8(b) the customary plot of  $\varphi_p$  and  $\varphi_n$  versus  $x$  is shown. If we neglect the effect of the series resistance terms denoted by  $R_1$  in Section 3, the change  $\delta\varphi$  will

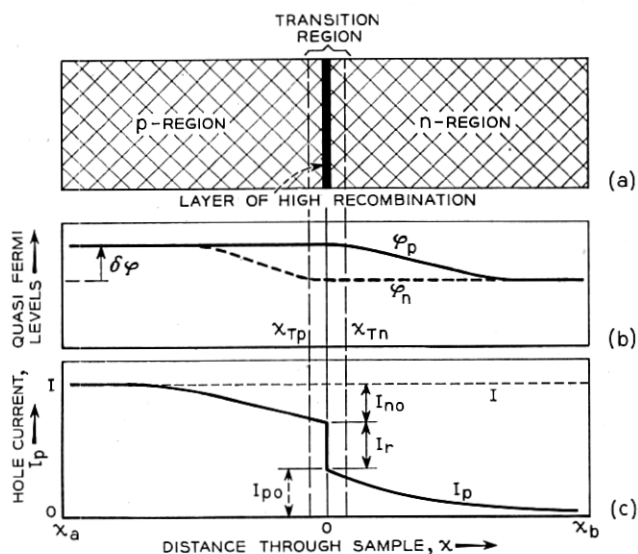


Fig. 8—The effect of a localized layer of high recombination rate on the junction characteristic.

- (a) Location of layer of high recombination rate.
- (b) Quasi Fermi levels.
- (c) Distribution of hole current showing rapid change at layer of high recombination rate.

occur in the  $p$ -region for  $\varphi_n$  and in the  $n$ -region for  $\varphi_p$ . The hole current flowing into the  $n$ -region will thus be the same as before and will be given by equation (4.11) or (4.18) and denoted by  $I_{p0}(\delta\varphi)$ . Similarly, the electron current will be  $I_{n0}(\delta\varphi)$ . In the layer we shall suppose that there is a rate of generation of hole electron pairs equal to  $g_a$  per unit area of the layer and a rate of recombination proportional to  $r_a n p$  per unit area. We suppose, furthermore, that the layer is so thin that  $n$  and  $p$  are uniform throughout the layer. The net rate of generation is thus

$$g_a - r_a n p = g_a [1 - e^{q(\varphi_p - \varphi_n)/kT}] \quad (4.34)$$

since for equilibrium conditions the rates balance so that  $r_a n_i^2 = g_a$ . The net hole current recombining in the layer per unit area is thus

$$I_r(\varphi_p - \varphi_n) = qg_a [e^{q(\varphi_p - \varphi_n)/kT} - 1] \quad (4.35)$$

There must, therefore, be a discontinuous decrease of hole current across the layer. The total hole current flowing in at  $x = x_a$ , which is also the total current  $I$ , thus does three things: for  $x < x_{Tp}$ , it combines with  $I_{n0}(\delta\varphi)$ ; for  $x_{Tp} < x < x_{Tn}$ , it combines with electrons at rate  $I_r(\delta\varphi)$ ; for  $x > x_{Tn}$ , it flows into the  $n$ -region in amount  $I_{p0}(\delta\varphi)$ . This leads to

$$I = I_{n0}(\delta\varphi) + I_{p0}(\delta\varphi) + I_r(\delta\varphi). \quad (4.36)$$

In other words the layer of high recombination acts like a rectifier in parallel with  $I_{n0}(\delta\varphi) + I_{p0}(\delta\varphi)$ . The frequency characteristic of  $I_r(\delta\varphi)$ , however, will be independent of frequency and will contribute a pure conductance to the admittance of the junction.

If the layer is considered to have finite width, however, it will exhibit frequency effects just as does  $I_p$  in the  $n$ -region. In Appendix III, we treat a case in which the layer is a part of the  $n$ -region itself but has a recombination time different from the main layer. If the time is shorter, a large amount of the hole current may recombine in this layer. For high frequencies, the current may not penetrate the layer, in which case the admittance for hole current is determined by the thin layer rather than by the whole  $n$ -type region. A case of this sort is shown in Fig. 7. In this case the thickness of the layer is  $\frac{1}{3}$  of its diffusion length and in it the lifetime of a hole  $\tau_l$  is  $\frac{1}{3}$  the value  $\tau_p$  in the main body of the  $n$ -region. The hole current will thus be restricted to this layer when the diffusion distance  $\sqrt{D/\omega}$  is less than the layer thickness ( $\frac{1}{3}$ )  $\sqrt{D\tau_l}$ ; this corresponds to  $\omega\tau_l > 9$  or  $\omega\tau_p > 81$ . The presence of the high rate of combination in the layer is evidenced by the tendency of  $G$  to be greater than  $S$  at high frequencies. If the layer were infinitely thin, as discussed above, it would simply add a constant conductance to the admittance.

#### 4.7 Patch Effect in $p$ - $n$ Junctions

If there are localized regions of high recombination rate, a "patch effect" may be produced in an  $n$ - $p$  junction. As an extreme example, suppose the value of  $g_a$  for the layer just considered is allowed to become very large; then the recombination resistance may become small compared to  $R_1$  in Section 3 and the junction will become substantially ohmic. If the region of high rate of recombination is relatively small compared to the area of the rest of the junction, then the behavior of the junction as a whole may be regarded as being that due to two junctions in parallel. Over most of the area,

the currents will flow as if the patch were not present so that one component of the current will be that due to the uniform junction. In addition there will be current due to recombination and generation in the patch. The series resistance to the patch will be relatively high due to the constriction of the current paths. On the other hand, the value of  $I_r(\delta\phi)$  associated with the patch may be very high. Hence the current due to the patch will be that of a low impedance ideal rectifier in series with a high resistance; and if the ratio of impedances is high enough, such a series combination amounts essentially to an ohmic leakage path. Thus patches in the *p-n* junction will tend to introduce leakage paths and destroy saturation in the reverse direction.

An extreme example of a region of high rate of recombination would be a particle of metal making a non-rectifying contact to both *p*- and *n*-type germanium. Since holes and electrons are essentially instantly combined in a metal, the boundary condition at the metal surface would be equality of  $\varphi_p$  and  $\varphi_n$ . This would mean that near the metal particle,  $\varphi_p$  and  $\sigma_n$  could not differ by  $\delta\varphi$ , the condition required, over some parts of the junction at least, in order for ideal rectification to occur.

A common source of imperfection in *p-n* junctions arises from dirt or fragments on the surface which overlap the junction. Even if these do not actually constitute a short circuit across the junction, they may furnish patches of the sort discussed here and modify the junction characteristic.

#### 4.8 Final Comments

Another possible cause for frequency effects may be found in the trapping of holes or electrons.<sup>14</sup> When an added hole concentration is introduced into an *n*-region, a certain fraction of the holes will be captured by acceptors and later re-emitted or else recombined with electrons while trapped. Investigation of this process is given in Appendix VI. One interesting result is that the trapping of holes in a uniform *n*-region cannot produce an effective susceptibility (i.e.  $i\omega C$ ) in excess of the conductance, as can a retarding field.

Finally it should be remarked that important and significant variations of the conductivity in the *p*- and *n*-regions may be produced by hole or electron injection. Under these conditions, when the hole concentration approaches  $n_n$ ,  $\psi - \varphi_n$  will vary. Under these conditions  $R_1$  may be appreciably altered. These factors favor the *p-n* junction as a rectifier since they lead to a reduction of series resistance under conditions of forward bias and thus tend to improve the rectification ratio.

<sup>14</sup> Frequency dependent effects in  $\text{Cu}_2\text{O}$  rectifiers have been explained in this way by J. Bardeen and W. H. Brattain, personal communication.