DIRECT LINEAR TRANSFORMATION FROM COMPARATOR COORDINATES INTO OBJECT SPACE COORDINATES IN CLOSE-RANGE PHOTOGRAMMETRY*

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BIOGRAPHICAL SKETCH

Youssef Ibrahim Abdel-Aziz was born in Cairo, Egypt in 1940. He earned his B.Sc. degree in mining engineering from Cairo University in 1963. He was then employed as a teaching assistant in civil engineering (surveying) in Cairo, Alexandria, and Assiut universities in Egypt. In 1967 he earned his Master's degree in civil engineering (photogrammetry) from Cairo University. Since 1968, Mr. Abdel-Aziz has been pursuing graduate studies in photogrammetric and geodetic engineering at the University of Illinois, concurrent with his employment as a research assistant. Mr. Abdel-Aziz was awarded University of Illinois' fellowships in 1969-70 and again in 1970-71.

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ABSTRACT

A method for photogrammetric data reduction without the necessity for neither fiducial marks nor initial approximations for inner and outer orientation parameters of the camera has been developed. This approach is particularly suitable for reduction of data from non-metric photography, but has also

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distinct advantages in its application to metric photography. Preliminary fictitious data tests indicate that the approach is promising. Experiments with real data are underway.

1. INTRODUCTION

In analytical photogrammetry, measurements of image points are normally done on comparators. The transformation of comparator coordinates into object space coordinates is usually performed in two steps:

a) Transformation from comparator coordinates into image coordinates, and
b) Transformation from image coordinates into object space (ground) coordinates.

For the transformation from comparator coordinates into image coordinates, it is necessary to calibrate and measure fiducial marks. For the transformation from image coordinates into object space coordinates, an iterative solution is generally used, for which one needs initial approximations for the unknown parameters (elements of outer orientation and in some cases also elements of inner orientation of the camera).

In working with hand-held non-metric cameras, neither of the above two requirements are satisfied. In view of the ever increasing use of non-metric cameras in close-range photogrammetry, particularly in cases of medium to low accuracy requirements, it was deemed desirable to develop a method suitable for data reduction from non-metric photography.

The proposed method involves a direct linear transformation from comparator coordinates into object space coordinates. In a sense, it is a simultaneous solution for the two aforementioned transformations. Since the image coordinate system is not involved in the approach, fiducial marks are not needed. Furthermore, the method is a direct solution and does not involve initial approximations for the unknown parameters of inner and outer orientation of the camera.

The proposed method is thus particularly suitable for reduction of data in non-metric photography. When applied to metric photography, the proposed approach yields at least the same accuracy as the conventional methods, but is easier to program (no linearization necessary) and uses less computer memory and executing time.

2. MATHEMATICAL BASIS OF THE PROPOSED METHOD

As mentioned above, the proposed method involves a simultaneous solution of two transformations which are usually done separately in conventional analytical photogrammetry.

The transformation of comparator coordinates into image coordinates is generally done in the following form:

$$\left.\begin{aligned} \bar{x} &= a_1 + a_2 x + a_3 y \\ \bar{y} &= a_4 + a_5 x + a_6 y, \end{aligned}\right\} \qquad (1)$$

where:

$\bar{x}$, $\bar{y}$ are image coordinates

x, y are comparator coordinates.

Such a transformation takes into account errors in perpendicularity between the x and y comparator coordinate axes, and possible differential linear distortions along the x and y comparator coordinate axes (due to lens distortion, film deformation and comparator unadjustment).

The transformation from image coordinates into object space coordinates is usually done using the following equation:

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ -c \end{bmatrix} = \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{bmatrix} , \ldots \qquad (2)$$

where: $\bar{x}$, $\bar{y}$ are image coordinates of points,

X,Y,Z are object-space coordinates of points,

X_o, Y_o, Z_o are the object-space coordinates of exposure stations, c is the camera constant,

λ is a scale factor, and

a_{ij} are the coefficients of transformation.

Equation (2) may be expressed as;

$$\left.\begin{aligned} &\bar{x} + c\ \frac{a_{11}(X-X_o)+a_{12}(Y-Y_o)+a_{13}(Z-Z_o)}{a_{31}(X-X_o)+a_{32}(Y-Y_o)+a_{33}(Z-Z_o)} = 0, \\ &\text{and} \\ &\bar{y} + c\ \frac{a_{21}(X-X_o)+a_{22}(Y-Y_o)+a_{23}(Z-Z_o)}{a_{31}(X-X_o)+a_{32}(Y-Y_o)+a_{33}(Z-Z_o)} = 0. \end{aligned}\right\} \quad (3)$$

Substituting equation (1) into equation (3) one gets:

$$\left.\begin{aligned} &a_1+a_2x+a_3y+c\ \frac{a_{11}(X-X_o)+a_{12}(Y-Y_o)+a_{13}(Z-Z_o)}{a_{31}(X-X_o)+a_{32}(Y-Y_o)+a_{33}(Z-Z_o)} = 0\ , \\ &\text{and} \\ &a_4+a_5x+a_6y+c\ \frac{a_{21}(X-X_o)+a_{22}(Y-Y_o)+a_{23}(Z-Z_o)}{a_{31}(X-X_o)+a_{32}(Y-Y_o)+a_{33}(Z-Z_o)} = 0\ . \end{aligned}\right\} \quad (4)$$

Eliminating y from equations (4), one gets:

$$(a_1 a_6 - a_4 a_3) + (a_2 a_6 - a_5 a_3) x +$$

$$c \frac{(a_{11} a_6 - a_3 a_{21})(X - X_o) + (a_{12} a_6 - a_{22} a_3)(Y - Y_o) + (a_{13} a_6 - a_{23} a_3)(Z - Z_o)}{a_{31}(X - X_o) + a_{32}(Y - Y_o) + a_{33}(Z - Z_o)} = 0. \quad \text{(5-a)}$$

Eliminating x from equations (4) one gets;

$$(a_1 a_5 - a_4 a_2) + (a_3 a_5 - a_6 a_2) y +$$

$$c \frac{(a_{11} a_5 - a_{21} a_2)(X - X_o) + (a_{12} a_5 - a_{22} a_2)(Y - Y_o) + (a_{13} a_5 - a_{23} a_2)(Z - Z_o)}{a_{31}(X - X_o) + a_{32}(Y - Y_o) + a_{33}(Z - Z_o)} = 0. \quad \text{(5-b)}$$

Equations (5-a) and (5-b) may be expressed as:

$$\left.\begin{aligned} & d_1 + d_2 x + \frac{b_1 X + b_2 Y + b_3 Z + b_4}{b_9 X + b_{10} Y + b_{11} Z + b_{12}} = 0, \\ & \text{and} \\ & d_3 + d_4 y + \frac{b_5 X + b_6 Y + b_7 Z + b_8}{b_9 X + b_{10} Y + b_{11} Z + b_{12}} = 0. \end{aligned}\right\} \quad (6)$$

Eliminating d_1 and d_3 from equations (6), one gets:

$$d_2 x + \frac{b_1'X+b_2'Y+b_3'Z+b_4'}{b_9'X+b_{10}'Y+b_{11}'Z+b_{12}'} = 0,$$

and

$$d_4 y + \frac{b_5'X+b'_6Y+b_9'Z+b_8'}{b_9'X+b_{10}'Yb_{11}'Z+b_{12}'} = 0. \qquad (7)$$

Eliminating d_2 and d_4 from equations (7), one gets:

$$x + \frac{b_1^*X+b_2^*Y+b_3^*Z+b_4^*}{b_9'X+b_{10}'Y+b_{11}'Z+b_{12}'} = 0,$$

and

$$y + \frac{b_5^*X+b_6^*Y+b_7^*Z+b_8^*}{b_9'X+b_{10}'Y+b_{11}'Z+b_{12}'} = 0. \qquad (8)$$

Eliminating b_{12}' from equations (8), one gets:

$$x + \frac{\ell_1X+\ell_2Y+\ell_3Z+\ell_4}{\ell_9X+\ell_{10}Y+\ell_{11}Z+1} = 0,$$

and

$$y + \frac{\ell_5X+\ell_6Y+\ell_7Z+\ell_8}{\ell_9X+\ell_{10}Y+\ell_{11}Z+1} = 0. \qquad (9)$$

Equations (9) are the basis of the proposed method.

3. MATHEMATICAL MODEL IN THE CONVENTIONAL (COLLINEARITY) APPROACH

As mentioned above, the transformation from comparator coordinates (x, y) into image coordinates $(\bar{x}, \bar{y})$ is usually done using equations (1):

$$\left.\begin{aligned} \bar{x} &= a_1+a_2x+a_3y \\ \bar{y} &= a_4+a_5x+a_6y. \end{aligned}\right\} \quad (1)$$

Since the selection of the image coordinate axes is arbitrary, let us select the definition shown in Fig. 1, where the $\bar{y}$ image coordinate axis is parallel to the y comparator coordinate axis and passes through the image principal point (0). The $\bar{x}$ image coordinate axis is perpendicular to the $\bar{y}$ axis and intersects it at the image principal point.

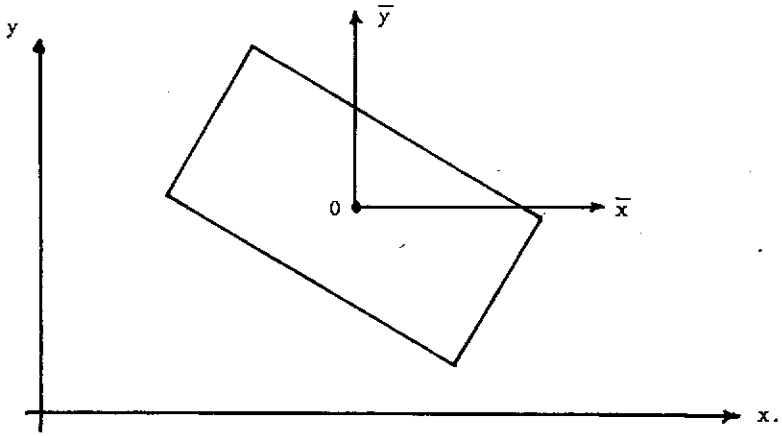

Fig. 1 -- Coordinate Axes.
(x & y; comparator coordinate axes; $\bar{x}$ & $\bar{y}$; image coordinate axes).

In this case, a_5 in equations (1) becomes zero and the relationship between comparator coordinates and image coordinates can be expressed as:

$$\left.\begin{aligned} \bar{x} &= a_1+a_2x+a_3y \\ \bar{y} &= a_4+a_6y \end{aligned}\right\} \quad (10)$$

Combining equations (10) and (3) one gets

$$\left.\begin{aligned} a_1+a_2x+a_3y - c\cdot\frac{a_{11}(X-X_0)+a_{12}(Y-Y_0)+a_{13}(Z-Z_0)}{a_{31}(X-X_0)+a_{32}(Y-Y_0)+a_{33}(Z-Z_0)} &= 0 \\ a_4+a_6y - c\cdot\frac{a_{21}(X-X_0)+a_{22}(Y-Y_0)+a_{23}(Z-Z_0)}{a_{31}(X-X_0)+a_{32}(Y-Y_0)+a_{33}(Z-Z_0)} &= 0 \end{aligned}\right\} \quad (11)$$

Equation (11) has 12 unknowns, but they are not linearly independent. These unknowns can be reduced to 11 linearly independent unknowns by eliminating a_2 and a_6 and introducing two unknowns C_x and C_y to replace C;

($C_x = \frac{c}{a_2}$; $C_y = \frac{c}{a_6}$, C_x and C_y reflect possible differential

linear distortions along x and y comparator axes). Equation (11) can thus be rewritten as:

$$\left.\begin{aligned} \bar{a}_1+x+\bar{a}_2y - C_x\frac{a_{11}(X-X_0)+a_{12}(Y-Y_0)+a_{13}(Z-Z_0)}{a_{31}(X-X_0)+a_{32}(Y-Y_0)+a_{33}(Z-Z_0)} &= 0 \\ \bar{a}_3+y-C_y\frac{a_{21}(X-X_0)+a_{22}(Y-Y_0)+a_{23}(Z-Z_0)}{a_{31}(X-X_0)+a_{32}(Y-Y_0)+a_{33}(Z-Z_0)} &= 0 \end{aligned}\right\} \quad (12)$$

Equations (12) represent the basic equations in the conventional (collinearity) approach. As explained above, these equations take into consideration the non-perpendicularity between comparator axes, and differential linear distortions along x and y axes.

4. OBSERVATION EQUATIONS

Expanding equations (12) by Taylor's series and neglecting second and higher order terms, one gets:

$$\left.\begin{aligned}
&V_x+a_{1y}V_y+b_{1x}\Delta\omega+b_{2x}\Delta\phi+b_{3x}\Delta\kappa+b_{4x}\Delta X_0+b_{5x}\Delta Y_0+b_{6x}\Delta Z_0+b_{7x}\Delta C_x+b_{8x}\Delta C_y\\
&+b_{9x}\Delta\bar{a}_1+b_{10x}\Delta\bar{a}_2+b_{11x}\Delta\bar{a}_3+F_x{}^{\circ}=0\\
&\text{and}\\
&V_y+b_{1y}\Delta\omega+b_{2y}\Delta\phi+b_{3y}\Delta\kappa+b_{4y}\Delta X_0+b_{5y}\Delta Y_0+b_{6y}\Delta Z_0+b_{7y}\Delta C_x+b_{8y}\Delta C_y\\
&+b_{9y}\Delta\bar{a}_1+b_{10y}\Delta\bar{a}_2+b_{11y}\Delta\bar{a}_3+F_y{}^{\circ}=0
\end{aligned}\right\}\quad(13)$$

where

V_x, V_y are errors in x and y

$a_{1y} = \bar{a}_2$, is partial derivative of F_x w.r.t. y

b_{1x}, b_{1y} are the partial derivatives of F_x and F_y (see footnote on following page) w.r.t. ω

b_{2x}, b_{2y} are the partial derivatives of F_x and F_y w.r.t. ϕ

b_{3x}, b_{3y} are the partial derivatives of F_x and F_y w.r.t. κ
b_{4x}, b_{4y} " " " " " " " " " X_o
b_{5x}, b_{5y} " " " " " " " " " Y_o
b_{6x}, b_{6y} " " " " " " " " " Z_o
b_{7x}, b_{7y} " " " " " " " " " C_x
b_{8x}, b_{8y} " " " " " " " " " C_y
b_{9x}, b_{9y} " " " " " " " " " a_1
b_{10x}, b_{10y} " " " " " " " " " $\bar{a}_2$
b_{11x}, b_{11y} " " " " " " " " " $\bar{a}_3$

F_x° and F_y° are functions of approximate values of the unknown parameters.

Equations (13) represent the observation equations in the conventional collinearity approach. The observation equations in the proposed direct approach may be obtained by expanding equations (9) and including all the zero terms (e.g. $0\ell_5$ and $0\ell_2$) for ease of reference:

$$F_x = \bar{a}_1 + x + \bar{a}_2 y - C_x \cdot \frac{a_{11}(X-X_0)+a_{12}(Y-Y_0)+a_{13}(Z-Z_0)}{a_{31}(X-X_0)+a_{32}(Y-Y_0)+a_{33}(Z-Z_0)}$$

$$F_y = \bar{a}_3 + y + C_y \frac{a_{21}(X-X_0)+a_{22}(Y-Y_0)+a_{23}(Z-Z_0)}{a_{31}(X-X_0)+a_{32}(Y-Y_0)+a_{33}(Z-Z_0)}$$

$$\left.\begin{aligned} &w_1 v_x + X\ell_1 + Y\ell_2 + Z\ell_3 + \ell_4 + 0\ell_5 + 0\ell_6 + 0\ell_7 + 0\ell_8 + xX\ell_9 + xY\ell_{10} + xZ\ell_{11} + x = 0, \\ &\text{and} \\ &w_2 v_y + 0\ell_1 + 0\ell_2 + 0\ell_3 + 0\ell_4 + X\ell_5 + Y\ell_6 + Z\ell_7 + \ell_8 + yX\ell_9 + yY\ell_{10} + yZ\ell_{11} + y = 0 \end{aligned}\right\} (14)$$

In equations (14) the factor w_1 and w_2 may be considered as weight factors, and their value can be easily determined in the solution.

A comparison between equations (13) and equations (14) indicate the simplicity of the proposed solution.

5. ANALYSIS OF ERRORS

Both the conventional and proposed approaches are influenced by the following errors:

a. Uncertainties in comparator measurements and errors in object space coordinates of control points.
b. Errors in mathematical modeling of film and lens distortions (random errors as well as unrepresented--or residual--systematic errors).

In addition, the conventional iterative approach is subject to computational errors due to:

a. Iteration criteria
b. Neglecting of second and higher terms in the linearization of the observation equations (13).

Obviously, the proposed direct solution is not subject to these computational errors.

6. FICTITIOUS DATA TESTS

A number of fictitious data tests were conducted to assess the capabilities of the proposed solution (equations 9) and compare them to the capabilities of the conventional approach (equations 12). As a datum for comparison of the two approaches, data from the collinearity approach with 9 unknowns (only parameters of inner and outer orientation are included, errors due to comparator unadjustment, lens distortion, and film deformation are not considered) were used. The tests covered the following aspects: handling of differential linear distortions along the x and y comparator coordinate axes, correction for non-perpendicularity of the comparator axes, accuracy of determination of the unknowns (standard error of unit weight), and computer executing time.

Tables I through V summarize the results of these preliminary tests.

Number of Iterations in Collinearity Approach	Number of Points Involved	Executing Time (secs)	
		Direct	Collinearity
2	43	6.16	12.56
4		6.16	22.14
5		6.16	27.21
No Convergence		6.16	No Solution
2	12	3.77	5.80
4		3.77	9.67
6		3.77	13.56
No Convergence		3.77	No Solution

Table I -- Computer Executing Time

The number of iterations indicated in the first column reflect the quality of approximations used in the various experiments with the collinearity solution.

Number of Points	σ_0 (μm)		
	Input	output	
		Direct Method	Conventional
43	3.000	2.781	2.995
	5.000	4.635	4.992
	10.000	9.271	9.985
	20.000	18.545	19.969
12	3.000	2.435	2.692
	10.000	8.117	8.972

Table II -- Standard Error of Unit Weight σ_0

Image coordinates perturbed by a random error of normal distribution with the standard errors indicated in the column "Input" and mean zero.

Method	Number of Control Points	Mean Square Error* (µm)			Estimated Standard Error of Unit Weight (µm)
		X	Y	Z	
Direct	5	NO SOLUTION POSSIBLE			
Collinearity	5				
Direct	6	205	179	408	1.770
Collinearity	6	200	174	407	2.005
Direct	10	165	135	334	2.293
Collinearity	10	172	137	353	2.261
Direct	20	157	94	342	2.792
Collinearity	20	161	95	356	2.744
Direct	30	138	100	301	2.888
Collinearity	30	141	101	310	2.892
Direct	43	135	94	297	2.782
Collinearity	43	135	92	303	2.995

Table III -- Accuracy of Object-Space Coordinates Obtained by the Direct and the Collinearity Approaches for different numbers of control points. Image coordinates perturbed by a random error of normal distribution with standard error: 3.000 µm and mean zero. Z-coordinate axis along camera axis. Colinearity according to equation 12. (* Mean square error =

$$\sqrt{\frac{\text{Sum of squares of residual errors}}{\text{number of points - 1}}}\ ).$$

Angle between x and y	Estimated Standard Error of Unit Weight (μm)		
	Collinearity I*	Collinearity II**	Direct Method
90°	2.970	2.995	2.781
91°	19.616	2.995	2.781
95°	97.616	2.995	2.781
99°	194.096	2.995	2.781

Table IV -- The effect of nonperpendicularity of the x and y comparator axes. Image coordinates perturbed by a random error of normal distribution with standard error: 3.000 μm, and mean zero. (* Collinearity I:9 unknowns, Collinearity II, 11 unknowns).

Scale Factor		Estimated Standard Deviation of Unit Weight (μm)		
x	y	Collinearity I**	Collinearity II**	Direct Method
1.000	1.000	2.970	2.995	2.781
1.000	1.0001	3.234	2.995	2.781
1.0001	1.0001	2.970	2.995	2.781
1.000	1.0002	3.896	2.995	2.781
1.0002	1.0002	2.970	2.995	2.781

Table V -- The effect of systematic differential linear distortion along the x and y comparator axes. Image coordinates perturbed by a random error of normal distribution with standard error: 3.000 μm and mean zero. (*Collinearity I: 9 unknowns, **Collinearit y II: 11 unknowns.)

Extensive testing of the proposed method using real data is currently underway.

7. SUMMARY OF COMPARISONS BETWEEN THE PROPOSED AND THE CONVENTIONAL APPROACHES

The proposed approach is particularly suitable for non-metric photography, where no fiducial marks are available, and can also be applied with distinct advantages for data reduction in metric photography. Following are some comments comparing the proposed approach to the conventional collinearity approach.

a. The proposed method yields at least the same accuracy as the conventional solution.
b. The proposed approach is a direct solution involving no iterations and needs no initial approximations for the unknowns. Thus a solution is obtained even in cases where the conventional collinearity approach fails due to the lack of reasonable approximations for the unknown parameters (inner and/or outer orientation elements). A case in point here would be metric photography for which the outer orientation is not known. It follows further that the proposed solution is not subject to computational errors due to iteration criteria nor to errors due to neglecting of second and higher order terms in linearizing the observation equations.
c. The proposed solution is relatively easy to program since it does not involve partial derivatives of the coefficients of the observation equation.
d. The computer executing time and the computer memory used are less in the proposed method than in the conventional collinearity solution.
e. The number of unknowns in the proposed direct method is the same as in the conventional approach, i.e. 11 (eleven). Thus a minimum of 6 well distributed control points are needed for a solution.
f. The proposed method is at a disadvantage in case of low accuracy requirements where one can neglect the comparator calibration errors and lens and film distortions. In this case the collinearity approach will have 9 unknowns compared to the 11 unknowns of the proposed method.

8. CONCLUDING REMARKS

Preliminary fictitious data tests indicate that the proposed approach is promising. Even though the method was originally developed for data reduction in close-range non-metric photography, it can be used with distinct advantages in conjunction with close-range metric photography. Experiments with real data are underway, and it is planned to report on these tests in the near future.

9. REFERENCES

1. Hallert, B., 1960, "Photogrammetry". McGraw-Hill.
2. von Gruber, O., 1930, "Ferienkurs in Photogrammetrie" Verlag von Konrad Wittwer.